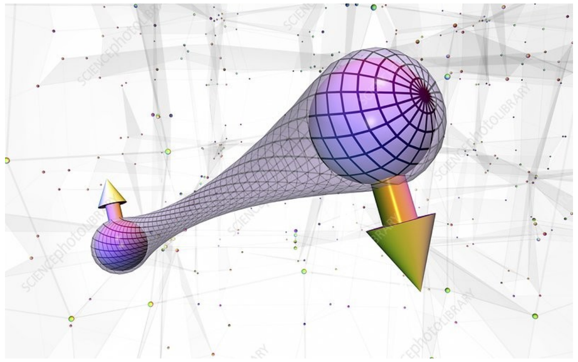
Quantum Entanglement and Bell Inequality Violation at High Energies



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Jan 21 2024

- “Entanglement” between two or more systems is a pure quantum phenomena
- It is induced by the interaction from which the entangled systems are produced
- Expected to violate Bell inequalities (set of correlation measurements)
- Violations incompatible with classical physics based on causality and local realism (locality) (EPR paradox, Local Hidden Variables Theories (LHVT))
- I will focus on quantum entanglement and Bell inequality violations within the Standard Model and that can be measured in real data
- Experiments involved: ATLAS, CMS, LHCb, BelleII, BESIII → BEPC

$$\underbrace{\text{ATLAS, CMS, LHCb}}_{\text{CERN}}$$

BelleII ↓ SuperKEK

$a|0\rangle|alive\rangle+b|1\rangle|dead\rangle$

“About your cat, Mr. Schrödinger—I have good news and bad news.”

What is entanglement ?

classical concept of phase space

In QM replaced by

by abstract Hilbert space

makes a gap in the description of composite systems

Consider multipartite system of ***n*** subsystems

- **Classical description** → Cartesian product of ***n*** subsystems → product of the ***n*** separate systems

- **Quantum description** → Hilbert space H → tensorial product of subsystem spaces

$$H = H_1 \otimes H_2 \otimes H_3 \otimes \cdots \otimes H_n$$

superposition principle

$$|\psi\rangle = \sum_{\mathbf{i}_n} c_{\mathbf{i}_n} |\mathbf{i}_n\rangle$$

$$|\mathbf{i}_n\rangle = |i_1\rangle \otimes |i_2\rangle \cdots |i_n\rangle$$

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \cdots |\psi_n\rangle$$

in general not possible to assign a single state vector to any of n subsystems

giving rise to the phenomenon of entanglement

Local realism

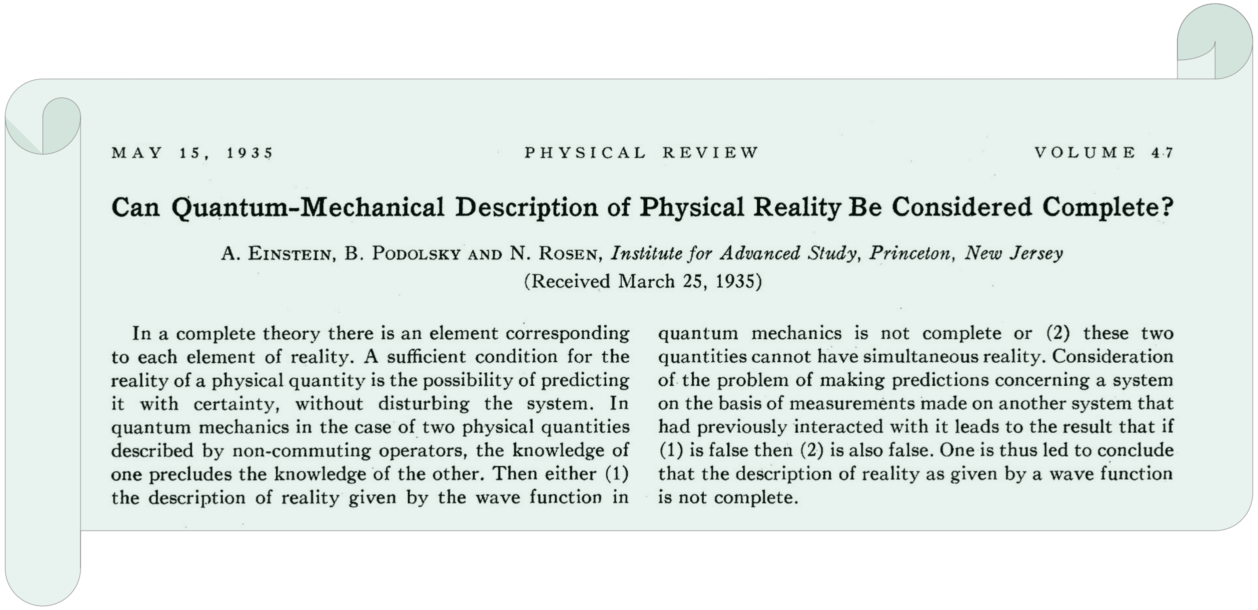
- Based on the (classical physics) idea that objects have definite properties whether or not they are measured

- and that measurements of these properties are not affected by events taking place sufficiently far away

- **Einstein Locality Principle**

Consider two systems **A** and **B** that have interacted in the past and are separated (space-like) far away

The results of a measurement on A is unaffected by operations on the distance system B



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

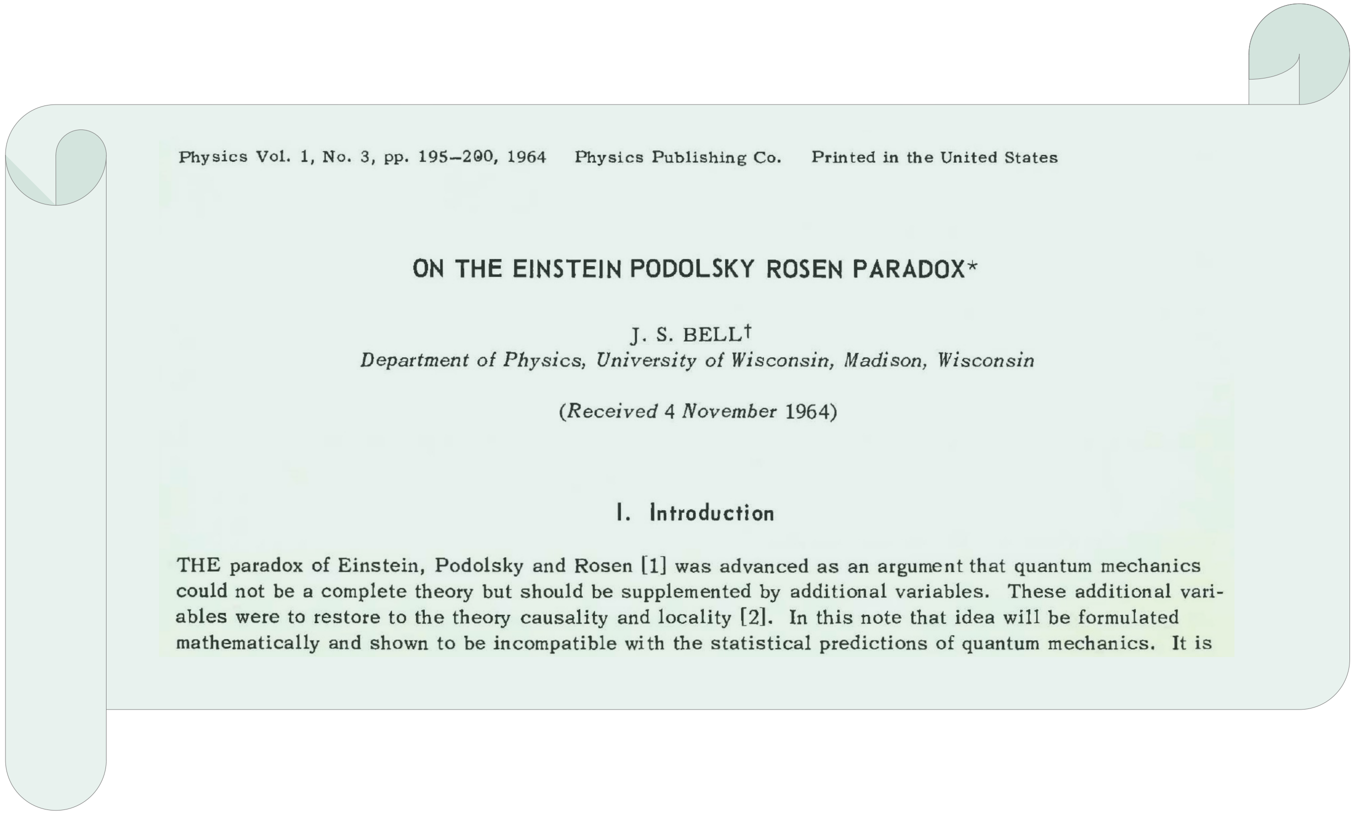
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

⇨ Based on locality principle they argue that QM is incomplete

- One may argue that the incompleteness of QM followed from EPR paradox is inherent in the probabilistic interpretation of Quantum Mechanics

- Dynamic behavior at microscopic level appears probabilistic only because some yet unknown parameters (hidden variables) have not been specified

- **Bell inequalities (1964):**
  a test to discriminate between local and non-local (QM) description of Nature



Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is

Quantum Entangled states violate Bell inequalities

Lets' explore this concept through an example:

- a muon pair created from the decay of a scalar particle (Higgs boson)
- the J=0 state is maximally entangled
- after being spatially separated, the spin of the two states are still entangled

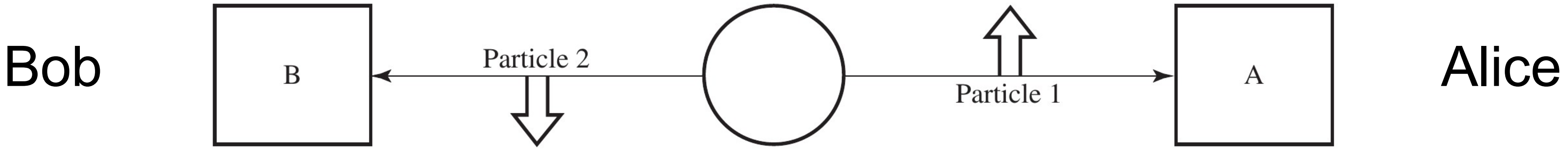
example

$$H \rightarrow \mu^+ + \mu^-$$

<u>*H* is a **J=0** state</u>

Maximum entangled state

Final state WF: $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\mu^+\uparrow;\mu^-\downarrow\rangle - |\mu^+\downarrow;\mu^-\uparrow\rangle\right]$

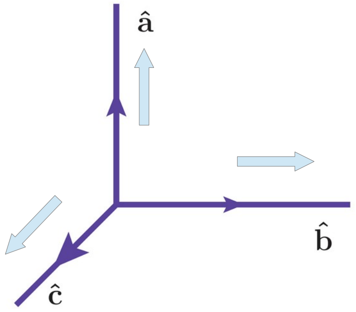


- measurement of spin in particle 1 induces correlation on spin measurement of particle 2
- measuring spin along same directions just test property of angular momentum conservation
- to check departure from Locality → require A and B to perform correlated measurements of spin-projection in **two different** directions

Not necessarily to be orthogonal

$$\{\hat{\mathrm{a}},\quad \hat{\mathrm{b}},\quad \hat{\mathrm{c}}\} \Rightarrow \quad \left[\mathbf{S}_{\hat{\mathbf{a}}},\mathbf{S}_{\hat{\mathbf{b}}}\right] \neq 0 \quad \left[\mathbf{S}_{\hat{\mathbf{a}}},\mathbf{S}_{\hat{\mathbf{c}}}\right] \neq 0 \quad \left[\mathbf{S}_{\hat{\mathbf{b}}},\mathbf{S}_{\hat{\mathbf{c}}}\right] \neq 0$$

Spin operators along directions **a,b**



Bell inequality

Alice **Bob**

$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\downarrow)$: spin $\uparrow$ along $\hat{\mathbf{a}}$ for particle 1 and spin $\downarrow$ along $\hat{\mathbf{b}}$ for particle 2

- **Locality assumption** → probability independence

$$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\downarrow)=P(\hat{\mathbf{a}}\uparrow;-)P(-;\hat{\mathbf{b}}\downarrow)$$

- If **A** measure S(**a**) and **B** measure S(**b**) there is a completely random correlation
- If **A** measure S(**a**) and **B** measure S(**a**) there is 100% (opposite) sign correlation between the two measurements due to angular momentum conservation
- If **A** makes **no measurements**, B's measurements show random results

Bell inequality

- Local deterministic theories (hidden variables) satisfiy **Bell inequality**

$$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\downarrow) = P(\hat{\mathbf{a}}\uparrow;-)P(-;\hat{\mathbf{b}}\downarrow)$$

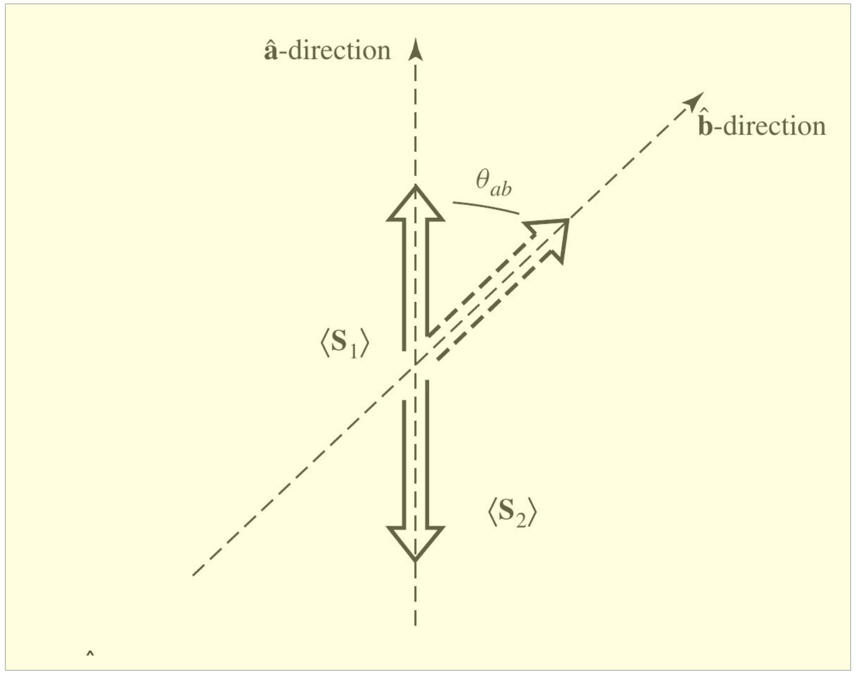
$$\boxed{P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\uparrow) \leq P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{c}}\uparrow) + P(\hat{\mathbf{c}}\uparrow;\hat{\mathbf{b}}\uparrow)}$$

(see backup slides for a proof)

- Compute these probability correlations in QM for an entangled S=0 state

QM predictions

- suppose observer A finds for particle 1 $\mathbf{S}_1 \cdot \hat{\mathbf{a}}$ to be positive (+) with certainty
- then particle 2 will be in a eigenstate of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ with negative (-) eigenvalue
- in order to compute $P(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+)$ we must consider a new quantization axis $\hat{\mathbf{b}}$



that makes an angle θ_{ab} with $\hat{\mathbf{a}}$

→ use the Wigner rotation matrix

- the probability that $\mathbf{S}_2 \cdot \hat{\mathbf{b}}$ measurement yields + when particle 2 is known to be in a eigenstate of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ is

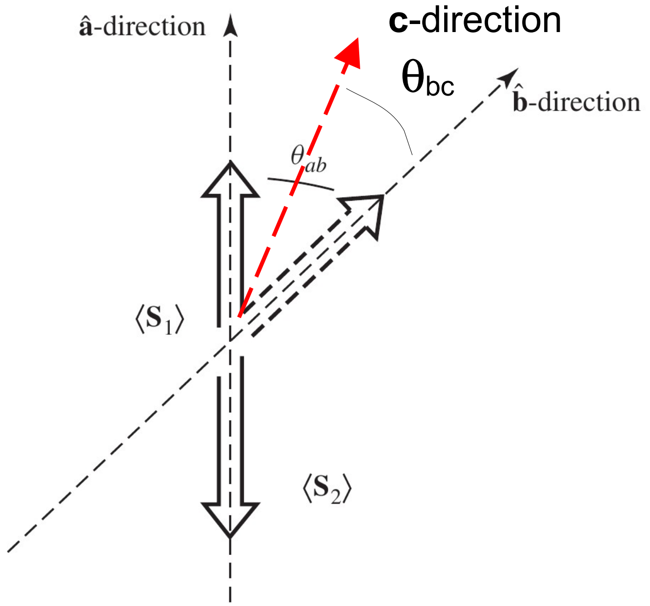
$$P(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+) = \left(\frac{1}{2}\right) \sin^2\left(\frac{\theta_{ab}}{2}\right)$$

factor ½ comes from the probability that $\mathbf{S}_1 \cdot \hat{\mathbf{a}}$ gives a + value

plug in into the Bell inequality and we get...

QM prediction of Bell inequality

$$\sin^2\left(\frac{\theta_{ab}}{2}\right) \leq \sin^2\left(\frac{\theta_{ac}}{2}\right) + \sin^2\left(\frac{\theta_{cb}}{2}\right)$$



choose for example

$$\theta_{ab} = 2\theta, \quad \theta_{ac} = \theta_{cb} = \theta$$

In this case Bell inequality is **violated** for

$$0 < \theta < \frac{\pi}{2}$$

- **optimization problem** → find directions where Bell inequality is maximally violated
- Maximum entangled states violate Bell inequalities but not provide the maximum violation in general

Bell inequality violation observed in entangled photons

QM is a non-local theory

measurements in A affects what will be measured in B, even if A and B are space-like separated apart, and no causal exchange of information between them is possible

The Nobel Prize in Physics 2022

© Nobel Prize Outreach. Photo: Stefan Bladh
Alain Aspect
Prize share: 1/3

© Nobel Prize Outreach. Photo: Stefan Bladh
John F. Clauser
Prize share: 1/3

© Nobel Prize Outreach. Photo: Stefan Bladh
Anton Zeilinger
Prize share: 1/3

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

[2] A. Zeilinger *et al.*, Nature **433**, 230 (2005)

[3] S.J. Freedman and J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972).
https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.28.938

[4] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982). A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49, **91** (1982)

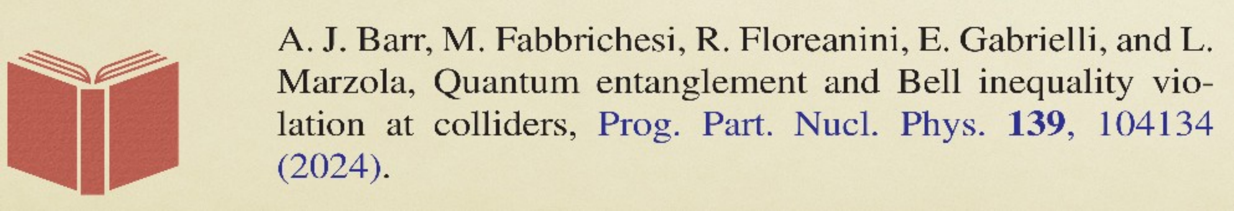
[5] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998). D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Nature **390**, 575 (1997)

[6] A. Aspect, Physics **8**, 123. (2015)

new challenge: testing entanglement and Bell inequality violation at high energies and in the presence of strong and weak interactions !

Quantifying entanglement and Bell inequality violation

- Requires the knowledge of the **Polarization Density Matrix** (PDM) of two-particles **A,B** production
- PDM can be fully reconstructed from the angular distributions of the single **A,B decay products** from data or a Montecarlo simulation (see backup slides for qubits and qutrits systems) in some specific basis → **quantum tomography**
- or analogously by measuring the complete set of **helicity amplitudes** from A,B decay products
- **but PDM can also be computed analytically** (using the SM theory) from the A,B polarizations
- knowledge of PDM allows to quantify (where possible) entanglement and Bell inequality violations
- use tools of quantum information theory



A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Quantum entanglement and Bell inequality violation at colliders, Prog. Part. Nucl. Phys. **139**, 104134 (2024).

the toolbox

Quantifying entanglement

difficult task → simplifies in the case of pure states

✓ Pure states (or pure ensables): the system is composed only by one quantum state $|\psi >$

Density matrix $\rho = |\psi\rangle\langle\psi|$ for pure states

✓ in real life we do not have pure states → but mixture of states $|\psi_n\rangle$ *n*=1,2,,...

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n| \ , \ \sum_n p_n = 1$$ ⇒ $$\mathrm{Tr}[\rho] = 1$$

$$\langle A\rangle = \mathrm{Tr}[\rho A]$$ → expectation value of operators (A) → can be computed in any base since Trace is base independent

✓ For pure states

$$\rho \doteq \begin{pmatrix} 0 & & & & & & & & & 0 \\ & 0 & & & & & & & & \\ & & \ddots & & & & & & & \\ & & & 0 & & & & & & \\ & & & & 1 & & & & & \\ & & & & & 0 & & & & \\ & & & & & & 0 & & & \\ & & & & & & & 0 & & \\ & & & & & & & & \ddots & \\ 0 & & & & & & & & & 0 \end{pmatrix}$$ (diagonal form)

⇒ $$\rho^2 = \rho$$

$$\mathrm{Tr}[\rho^2] = 1$$

Qubits

spin-1/2 particles, photon

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \frac{1}{4}\left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{ij} C_{ij} (\sigma_i \otimes \sigma_j)\right]$$

σ_i = 2 x 2 Pauli matrices

4 x 4 matrix

Polarization coefficients

spin-correlations coefficients

- ## Entanglement

(for pure states)

Concurrence

$$\mathscr{C}[|\psi\rangle] \equiv \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_A)^2\right]\right)} = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho_B)^2\right]\right)}$$

for general states →

$$\mathscr{C}[\rho] = \inf_{\{|\psi\rangle\}} \sum_i p_i\, \mathscr{C}[|\psi_i\rangle]$$

vanishes for separable states,

max value = 1

$$\rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right]$$

Trace performed in subsystem B

For qubits problem is solved:

$$R = \rho\,(\sigma_2 \otimes \sigma_2)\,\rho^*\,(\sigma_2 \otimes \sigma_2)$$

find → r_i square root of R eigenvalues, i=1,2,3,4 with r_1 be the largest one

$$\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$$

If > 0 it signals the presence of entanglement

C.H. Bennett, D.P. Divincenzo, J.A. Smolin, W.K. Wootters, Phys. Rev. A 54 (1996) 3824

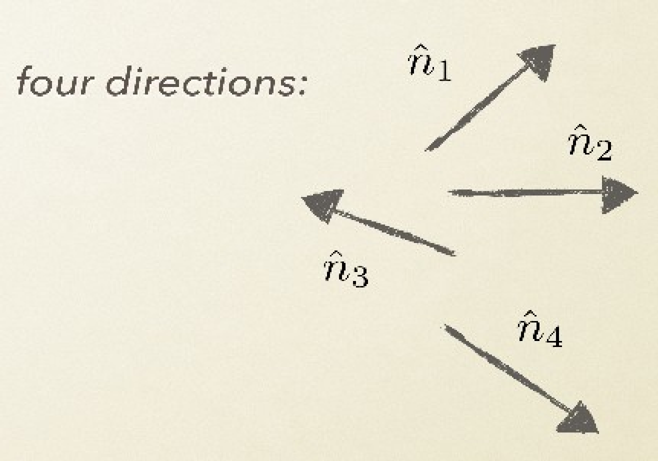
Qubits

Bell inequality violation

$\vec{n}_1,\ \vec{n}_3$ → for Alice ⇨ $(\hat{A}_1,\ \hat{A}_2)$

$\vec{n}_2,\ \vec{n}_4$ → for Bob ⇨ $(\hat{B}_1,\ \hat{B}_2)$

2 outcomes
(qubits)



CHSH inequality

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \leq 2$$

→ satisfied by LHVT

Clauser-Horne-Schimony-Holt
Phys. Rev. Lett. 24 (1970) 549

⇩

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

C_{ij} → **Correlation matrix**

$M = C^T C$ ⇨ $[m_1, m_2, m_3]$ eigenvalues ⇨ m_1 and m_2 the largest ones

→ optimization problem solved by the

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$ ⇨ Violation of Bell inequality

R. Horodecki *et al* , Phys. Lett. A200 5 (1995) 340

Qutrits

massive spin-1 particles

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \left(\frac{1}{9}\,[\mathbb{1}\otimes\mathbb{1}] + \sum_a f_a\,[\mathbb{1}\otimes T^a] + \sum_a g_a\,[T^a\otimes\mathbb{1}] + \sum_{ab} h_{ab}\left[T^a\otimes T^b\right]\right)_{\lambda_1\lambda_1',\lambda_2\lambda_2'}$$

T^a = 3 x 3 Gell-Mann matrices

9 x 9 matrix

● Entanglement

F. Mintert, A. Buchleitner, PRL 98 (2007) 140505

Concurrence

- Difficult to compute, no analytical solution exists for general qutrits states
- only lower bound available

$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho]$$

$$\begin{aligned} \mathscr{C}_2 \quad = \quad & 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,, \\ & -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]\,, \end{aligned}$$

Lower bound:
witness of Entanglement

$$0 \leq \mathscr{E}[\rho] \leq \ln d$$

Entropy

Valid only for pure states

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

d=2 for qubits
d=3 for qutrits

Qutrits

Bell inequality violation

$$\begin{aligned}\mathcal{I}_3 &= P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ &\quad - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)\end{aligned}$$

$(A_1 \ , \ A_2)$ $(B_1 \ , \ B_2)$ each can take values → $\{0, 1, 2\}$

$$\mathcal{I}_3 = \mathrm{Tr}\big[\rho\, \mathcal{B}\big]$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- **CGLMP** $\mathcal{I}_3 \leq 2$ ⇨ satisfied by HVLT

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett 88 (2002) 040404

- in order to maximize the violation of Bell inequality

$$\mathcal{B} \to (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

U,V are unitary 3x3 matrices
(depend on the kinematic of the process)

Bell inequality test at collider

Pioneering works

suggested here

N.A. Tornqvist, Found. Phys. 11 (1981) 171-177
N.A. Tornqvist, Phys. Lett. A 117 (1986) 1-4

Decays of charmonium states into a pair of entangled barions Λ

$$\eta_c,\ \chi_c \ \text{and}\ J/\psi \longrightarrow \Lambda + \bar{\Lambda}$$

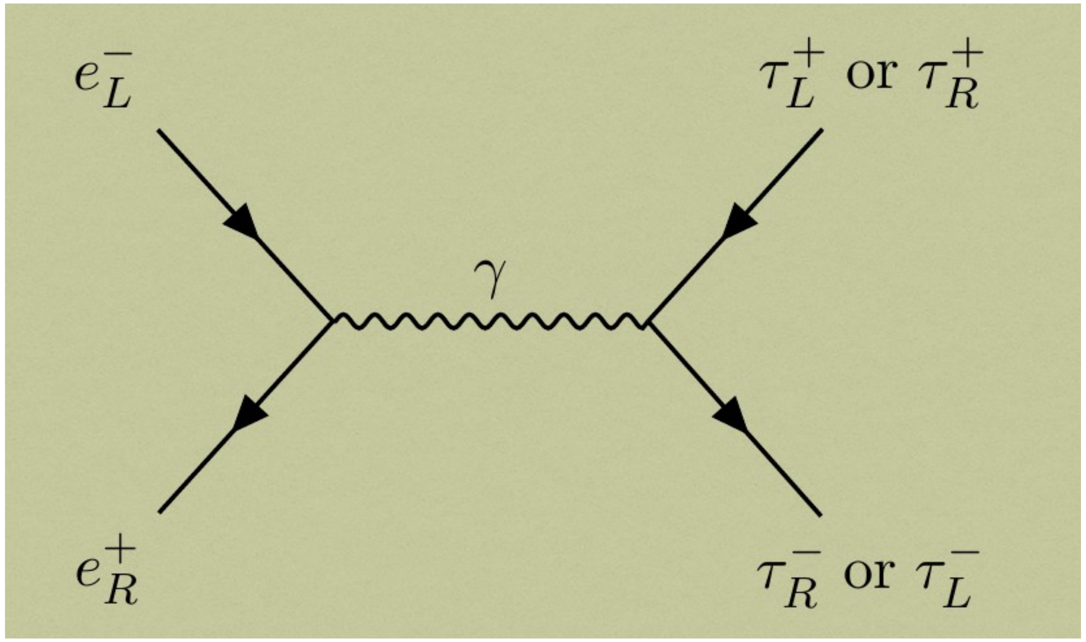
analyzed here

S.P. Baranov, J. Phys. G 35 (2008)
S.P. Baranov, Int. J. Mod. Phys. A 24 (2009) 480-483
S. Chen, Y. Nakaguchi, S. Komamiya, PTEP 2013 (6) (2013) 063A01

P. Privitera, Phys. Lett. B 275 (1992) 172-180

$$e^+e^- \rightarrow \tau^+\ \tau^-$$

example



in CM frame

massless limit

QM → non separable entangled states

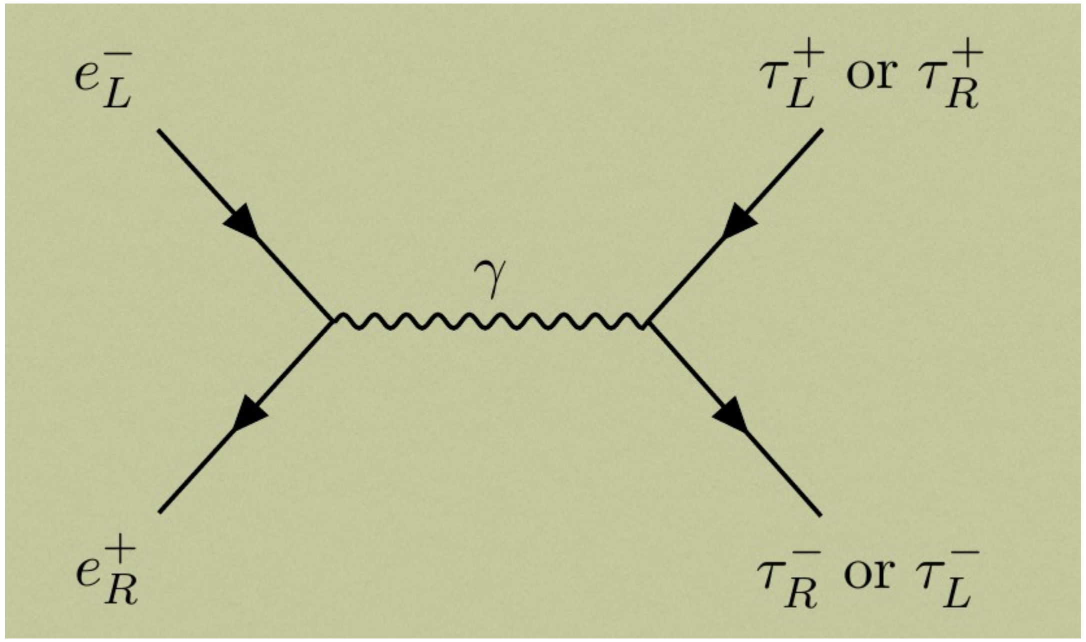
$$|\Psi\rangle = \xi_1 |\tau^-_L\rangle|\tau^+_L\rangle + \xi_2 |\tau^-_R\rangle|\tau^+_L\rangle + \xi_3 |\tau^-_L\rangle|\tau^+_R\rangle + \xi_4 |\tau^-_R\rangle|\tau^+_R\rangle$$

$$\left(\sum_i |\xi_i|^2 = 1 \right)$$

Deterministic theories → separable states (example)

$$|\Psi\rangle_{\rm cl} = |\tau^-_R\rangle|\tau^+_L\rangle \qquad \text{OR} \qquad |\tau^-_L\rangle|\tau^+_R\rangle$$

example



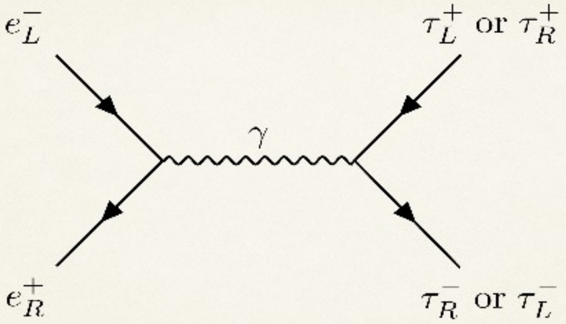
Θ scattering angle in the C.M. frame

relativistic massless limit

$$|\Psi\rangle = \underbrace{\Big(1+\cos\Theta\Big)}_{\xi_2 = D^{(1)}_{1,1}(\Theta)}|\tau_R^-\rangle|\tau_L^+\rangle + \underbrace{\Big(1-\cos\Theta\Big)}_{\xi_3 = D^{(1)}_{1,-1}(\Theta)}|\tau_L^-\rangle|\tau_R^+\rangle$$

Wigner D-matrix

$J=\pm1 \quad J_z=\pm1 \quad (\Theta=0)$	$\|\tau_R^-\rangle\,\|\tau_L^+\rangle$	separable
$J=\pm1 \quad J_z=0 \quad (\Theta=\pi/2)$	$\frac{1}{\sqrt{2}}\Big(\|\tau_R^-\rangle\,\|\tau_L^+\rangle+\|\tau_L^-\rangle\,\|\tau_R^+\rangle\Big)$	entangled (Bell state)

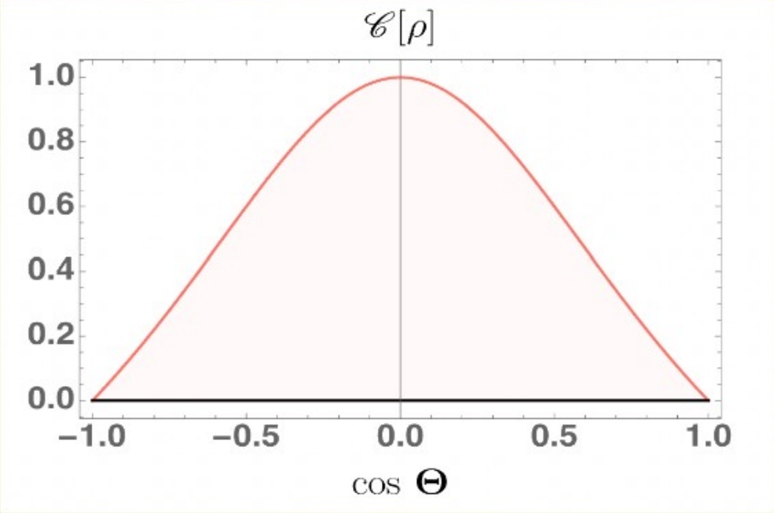


massless limit

$$\Big(1+\cos\Theta\Big)|\tau^-_R\rangle\,|\tau^+_L\rangle + \Big(1-\cos\Theta\Big)|\tau^-_L\rangle\,|\tau^+_R\rangle$$

Concurrence

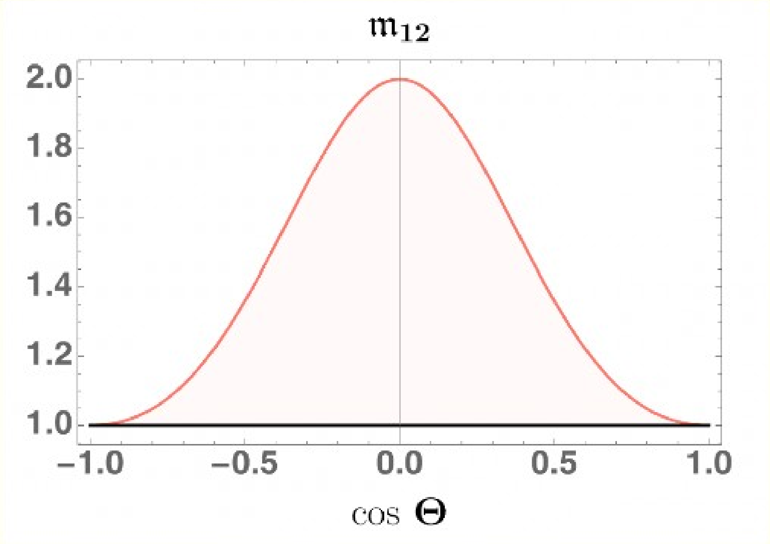
$$\mathcal{C}[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2\Theta}{1+\cos^2\Theta}$$



$$\Big(1+\cos\Theta\Big)|\tau^-_R\rangle\,|\tau^+_L\rangle + \Big(1-\cos\Theta\Big)|\tau^-_L\rangle\,|\tau^+_R\rangle$$

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4\Theta}{(1+\cos^2\Theta)^2}$$



courtesy of M. Fabbrichesi

Local deterministic models satisfy Bell inequality

Quantum mechanics does not

Both Entanglement and Bell inequality

can be studied at colliders

- high-energy regime
- in the presence of strong and weak interactions
- qubits and qutrits

Where have we already seen Entanglement or Bell inequality violation at high energies?

1

Flavor space

$K^0\bar{K}^0$ oscillations

✔

Probing CPT and T-reversal with entangled neutral Kaons

F. J. Bernabeu, A. Di Domenico, P. Villanueva, JHEP 10 (2015) 13
J. Bernabeu, A. Di Domenico, Phys. Rev. D 105, 116004 (2022)

Bell locality condition

$$p_\lambda(f_1,\tau_1;f_2,\tau_2)=p_\lambda(f_1,\tau_1;-,\tau_2)\;p_\lambda(-,\tau_1;f_2,\tau_2)$$

$\mathcal{P}(f_1,\tau_1;-,\tau_2) \rightarrow$ Probability of finding state f_1 at time τ_1

Bell inequality $$\mathcal{P}(f_1,\tau_1;f_2,\tau_2)-\mathcal{P}(f_1,\tau_1;f_4,\tau_2)+\mathcal{P}(f_3,\tau_1;f_2,\tau_2)+\mathcal{P}(f_3,\tau_1;f_4,\tau_2)$$
$$\leq \mathcal{P}(f_3,\tau_1;-,\tau_2)+\mathcal{P}(-,\tau_1;f_2,\tau_2)$$

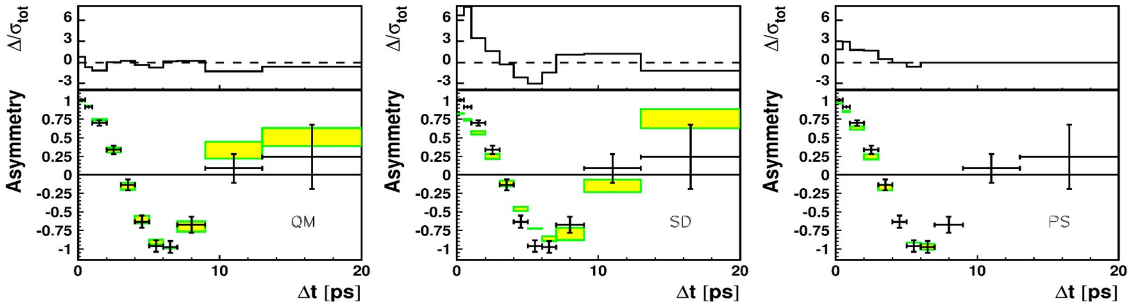
a non vanishing value of epsilon'/epsilon (direct CP violation) implies Bell inequality violation

F. Benatti, R. Floreanini Phys. Rev. D57 (1998); Eur. Phys. J C13 (2000) 267

$B^0\bar{B}^0$ oscillations

✔

$$|\psi\rangle=\frac{1}{\sqrt{2}}\left[|B^0\rangle_1\otimes|\overline{B}^0\rangle_2-|\overline{B}^0\rangle_1\otimes|B^0\rangle_2\right]$$



Asymmetry

$$A(\Delta t)=(R_{\mathrm{OF}}-R_{\mathrm{SF}})/(R_{\mathrm{OF}}+R_{\mathrm{SF}})$$

$$R_{\mathrm{OF/SF}}=\text{ rate of Opposite/Same}-\text{Flavor}$$

Data favour QM over spontaneous disentanglement at 13σ and over Pompili-Selleri model (LHVT) at 5.1σ

A Go, Belle Collaboration, Phys. Rev. Lett . 99 (2007) 131802

1

Flavor space

Neutrino oscillations

✔

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k}\,|\nu_k\rangle$$

flavor states

mass states

Leggett-Garg inequality violation

$$C_{ij} \equiv \langle \hat{Q}(t_i)\hat{Q}(t_j)\rangle$$

$$\hat{Q}(t) \equiv \hat{U}^\dagger(t)\hat{Q}\hat{U}(t)$$

$$K_n \equiv \sum_{i=1}^{n-1} C_{i,i+1} - C_{1,n}$$

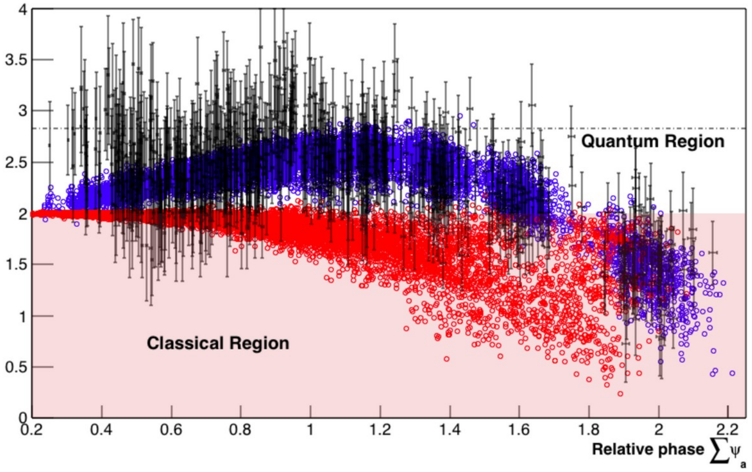
under hypothesis of realism and non-invasive measurements

$$K_n \leq n - 2$$

Minos (6σ)

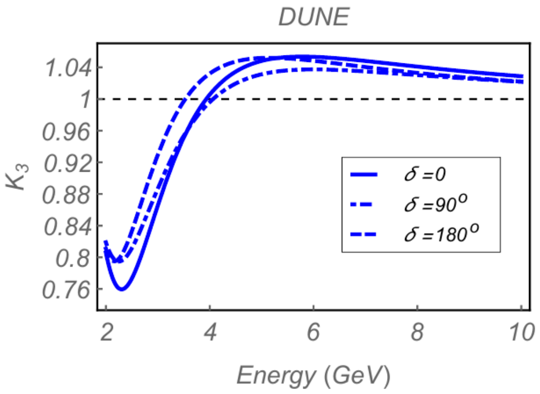
Dune, Nova, T2K

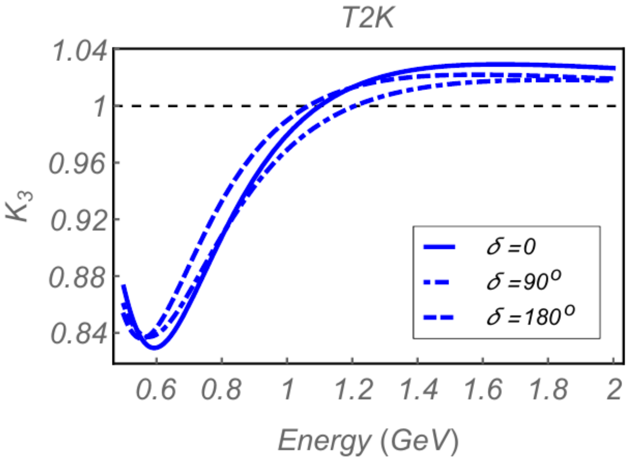
K_4



$K_4 < 2$

$K_3 < 1$





JA Formaggio, DI Kaiser, MM Murskyj and TE Weiss, Phys. Rev. Lett. 117 (2016) 050402

Violation of LG inequality occurs over a distance of 735km.

J Naikoo et al, Phys. Rev. D 99 (2019) 095001

2

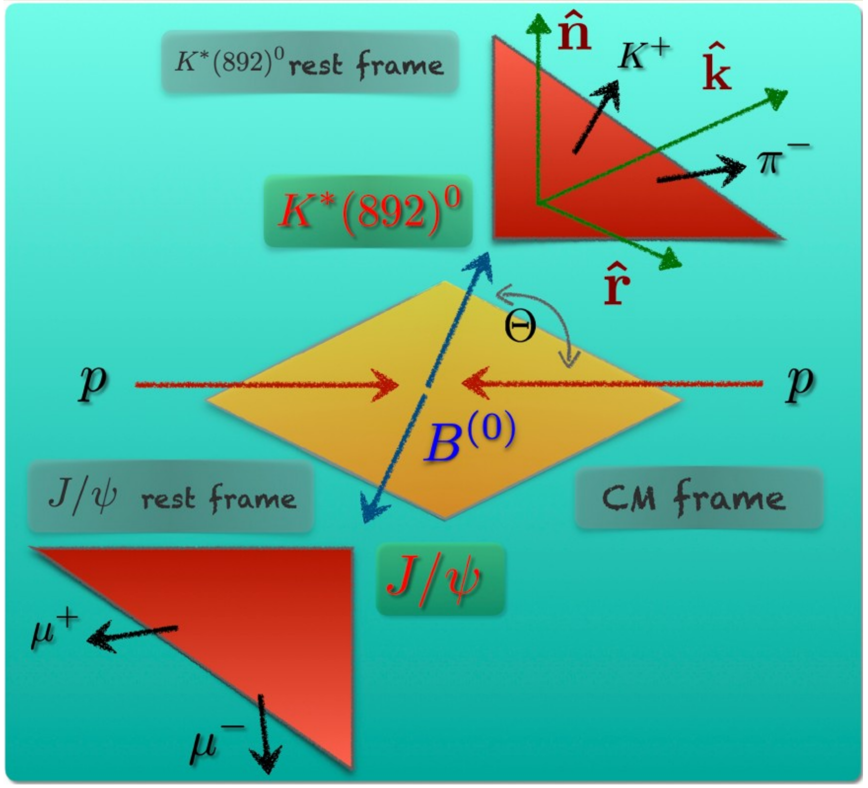
spin-1 qutrits

B meson decays $B \to V_1 V_2$

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\Big[h_+\,|\mathbf{V_1(+)V_2(-)}\rangle + h_0\,|\mathbf{V_1(0)V_2(0)}\rangle + h_-\,|\mathbf{V_1(-)V_2(+)}\rangle\Big]$$

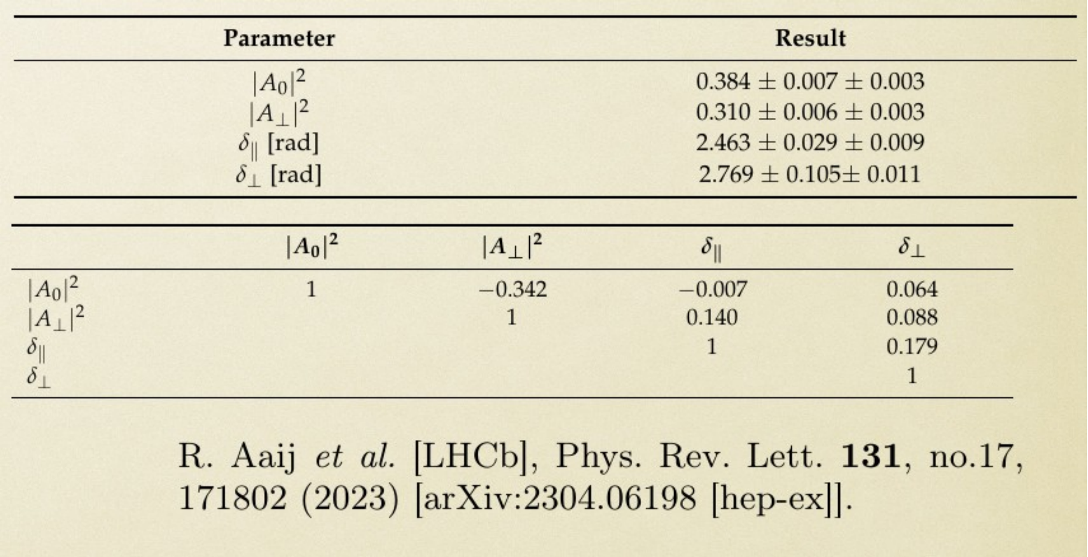
h_i = helicity amplitudes

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$



$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_0}{|H|} = A_0\,, \qquad \frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}} \qquad \text{and} \qquad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}$$



Parameter	**Result**
$\lvert A_0\rvert^2$	$0.384 \pm 0.007 \pm 0.003$
$\lvert A_\perp\rvert^2$	$0.310 \pm 0.006 \pm 0.003$
$\delta_\parallel$ [rad]	$2.463 \pm 0.029 \pm 0.009$
$\delta_\perp$ [rad]	$2.769 \pm 0.105 \pm 0.011$

	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\delta_\parallel$	$\delta_\perp$
$\lvert A_0\rvert^2$	1	−0.342	−0.007	0.064
$\lvert A_\perp\rvert^2$		1	0.140	0.088
$\delta_\parallel$			1	0.179
$\delta_\perp$				1

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023) [arXiv:2304.06198 [hep-ex]].

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104
EG and L. Marzola, Symmetry 6 (2024) 8, 1036

2

B meson decays

First time of direct measurement of Bell inequality violation at high energy !

	Entanglement	Bell inequality	Significance of Bell inequality violation
	$\mathcal{E}$	$\mathcal{I}_3$	
• $B^0 \to J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	36σ
• $B^0 \to \phi K^*(892)^0$ [18]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	
• $B^0 \to \rho K^*(892)^0$ [19]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$	
• $B_s \to \phi\phi$ [20]	0.734 ± 0.037	2.525 ± 0.064	8.2σ
• $B_s \to J/\psi\phi$ [21]	0.731 ± 0.032	2.462 ± 0.080	

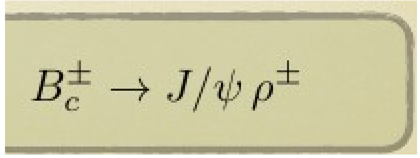
* → correlations uncertainties are missing → upperbound on error

Free from locality loophole

(see backup slides for details)

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104
EG and L. Marzola, Symmetry 6 (2024) 8, 1036

K. Chen et al, Eur. Phys. J. C 84 (2024) 580



$B_c^\pm \to J/\psi\, \rho^\pm$

3 Entanglement in pairs of top quarks

$$D = \frac{1}{3}\,\mathrm{Tr}\,C_{ij} \qquad \mathscr{C}[\rho] = \max[-1-3D,\,0]/2$$

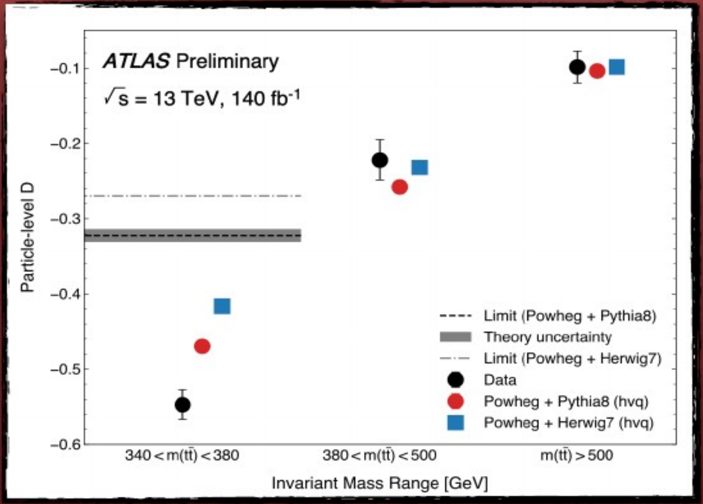
$D <$ -1/3 sufficient condition for entanglement

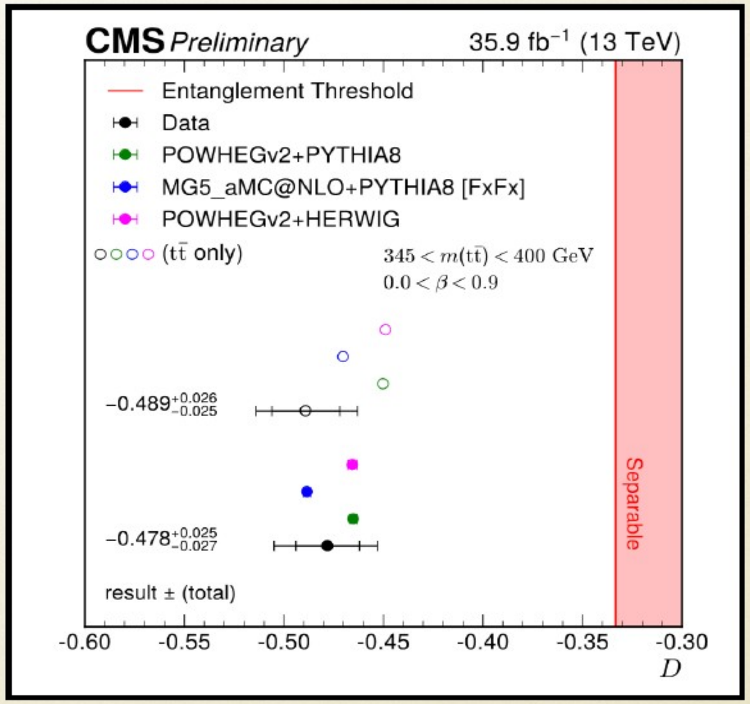
→ also sensitive to Toponium formation

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\Big(1 - D\cos\phi\Big)$$





Significance > 5σ

$$D = -0.478^{+0.025}_{-0.027}$$

$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, Nature 633 (2024) 542

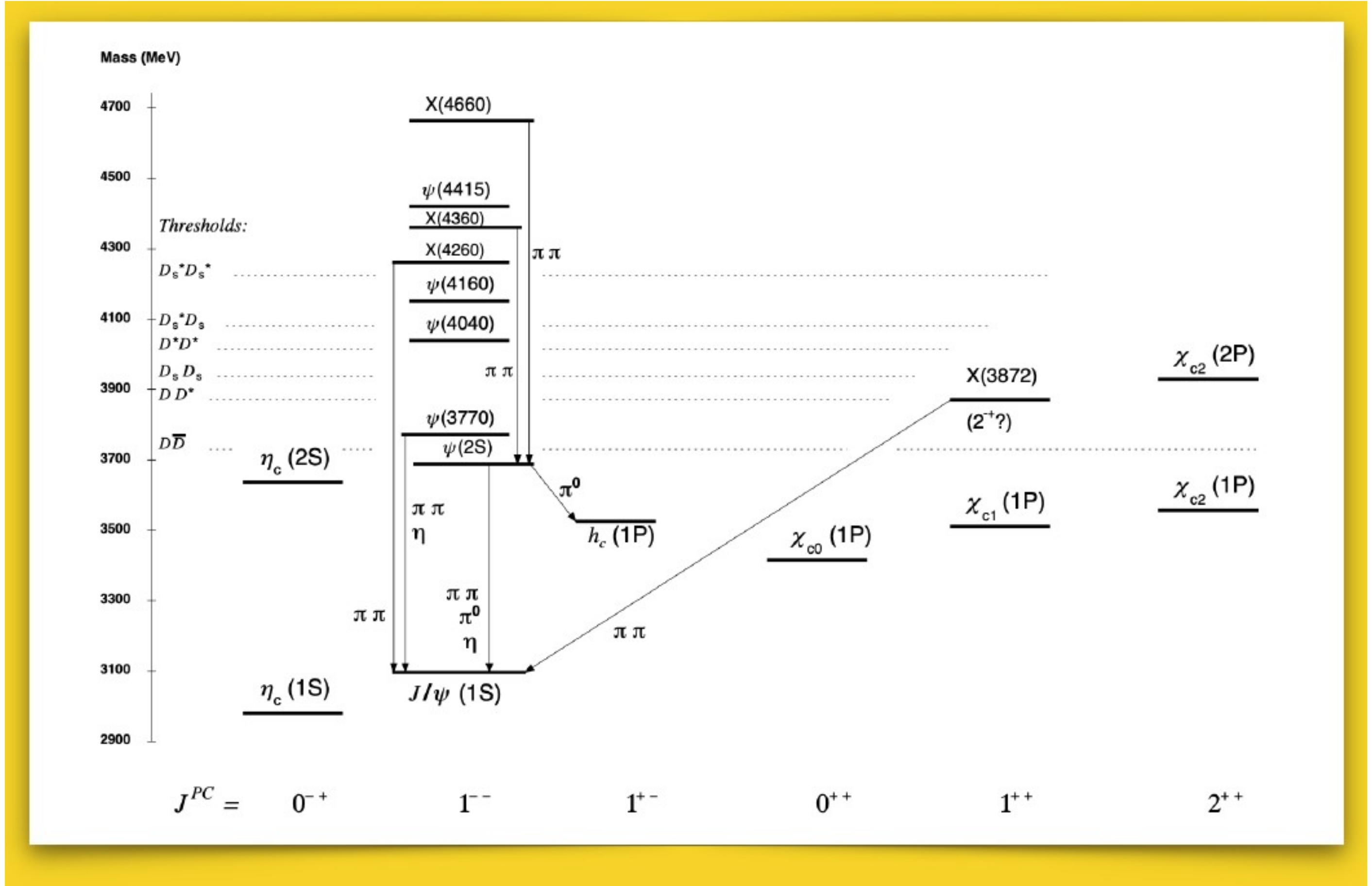
✔

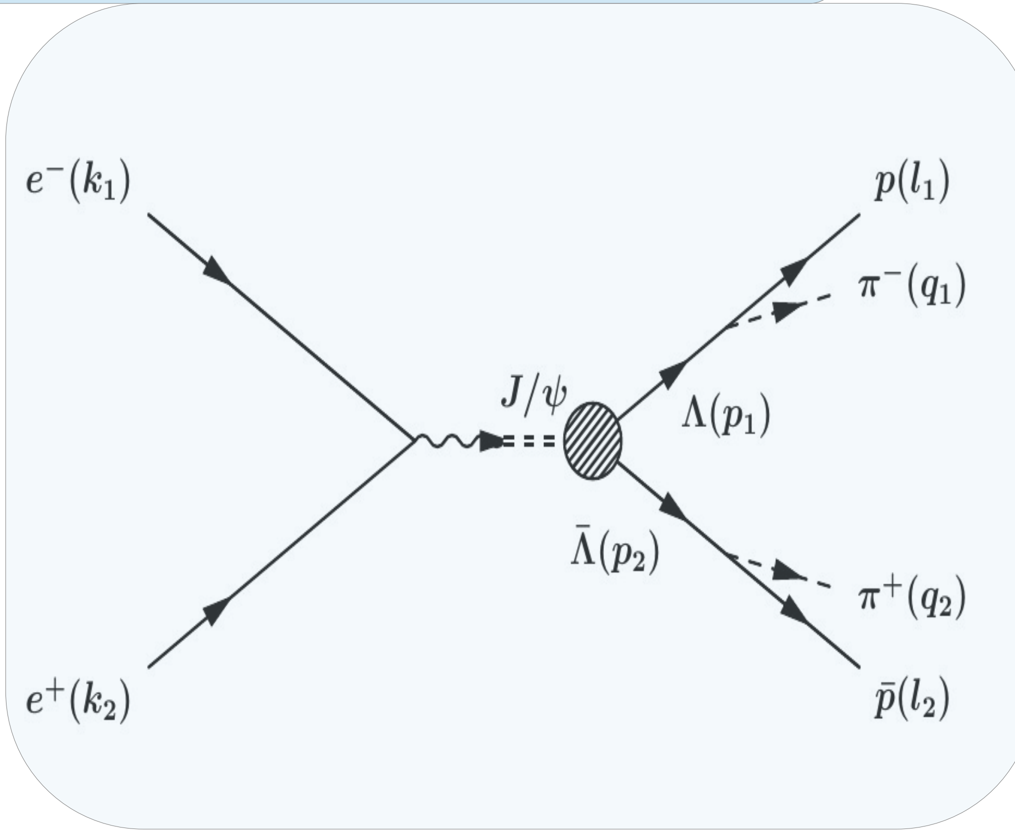
CMS Collaboration, arXiv:2406.03976 (2024)
CMS Collaboration, arXiv:2409.11067 (2024)

ϕ is the angle between the respective leptons as computed in the rest frame of the decaying top and anti-top

$340\,\text{GeV} < m_{t\bar{t}} < 380\,\text{GeV}$

Charmonium





$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p} \qquad \mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$$

$$\cos\Theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$$

scattering angle
in c.o.m frame

helicity amplitudes decomposition

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0) D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Wigner rotation D-matrix

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 110 (2024) 3, 053008
see, also: S. Wu et al. Phys Rev. D110 (2024) 054012

Qubits (spin ½)

$$\Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$$

$$\mathrm{d}\sigma \propto \mathcal{W}(\boldsymbol{\xi})\, \mathrm{dcos}\,\theta\, \mathrm{d}\Omega_1 \mathrm{d}\Omega_2$$

$$\boldsymbol{\xi} \;=\; (\theta, \Omega_1, \Omega_2)$$

$$\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) =& \mathcal{F}_0(\boldsymbol{\xi}) + \alpha \mathcal{F}_5(\boldsymbol{\xi}) \\
& + \alpha_1 \alpha_2 \left(\mathcal{F}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha^2} \cos(\Delta\Phi) \mathcal{F}_2(\boldsymbol{\xi}) + \alpha \mathcal{F}_6(\boldsymbol{\xi})\right) \\
& + \sqrt{1-\alpha^2} \sin(\Delta\Phi) \left(\alpha_1 \mathcal{F}_3(\boldsymbol{\xi}) + \alpha_2 \mathcal{F}_4(\boldsymbol{\xi})\right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_0(\boldsymbol{\xi}) =& 1 \\
\mathcal{F}_1(\boldsymbol{\xi}) =& \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2 \\
\mathcal{F}_2(\boldsymbol{\xi}) =& \sin\theta \cos\theta (\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2) \\
\mathcal{F}_3(\boldsymbol{\xi}) =& \sin\theta \cos\theta \sin\theta_1 \sin\phi_1 \\
\mathcal{F}_4(\boldsymbol{\xi}) =& \sin\theta \cos\theta \sin\theta_2 \sin\phi_2 \\
\mathcal{F}_5(\boldsymbol{\xi}) =& \cos^2\theta \\
\mathcal{F}_6(\boldsymbol{\xi}) =& \cos\theta_1 \cos\theta_2 - \sin^2\theta \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2. \qquad (6.56)
\end{aligned}$$

$$w_{\frac{1}{2}\frac{1}{2}} = w_{-\frac{1}{2}-\frac{1}{2}} = \frac{\sqrt{1-\alpha}}{\sqrt{2}}$$

$$w_{\frac{1}{2}-\frac{1}{2}} = w_{-\frac{1}{2}\frac{1}{2}} = \sqrt{1+\alpha}\, \exp[-i\Delta\Phi]$$

maximum likelihood fit

$$\alpha = 0.4748 \pm 0.0022|_{\rm stat} \pm 0.0031|_{\rm syst}$$

$$\Delta\Phi = 0.7521 \pm 0.0042|_{\rm stat} \pm 0.0066|_{\rm syst}$$

to extract helicity amplitudes

$$\Theta \equiv \theta$$

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0) D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Charmonium spin-0 states

Qubits (spin ½)

$$\eta_c \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \chi_c^0 \to \Lambda + \bar{\Lambda}$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$\rho_{\Lambda\,\Lambda} = |\psi_0\rangle\langle\psi_0| = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Concurrence **Horodecki condition**

$$\mathscr{C}[\rho] = 1 \qquad \mathfrak{m}_{12} = 2$$

- maximum violation of Bell inequality
- data not yet available to assess significance

N.A. Tornqvist, Phys. 11 (1981) 171-177
N.A. Tornqvist, Phys. Lett. A 117 (1986) 1 4
S.P. Baranov, Phys. G 35 (2008) 075002

Qutrits (spin 1)

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \ \pm 0.019|_{\text{syst}}$$

BesIII Collaboration, JHEP 05 (2023) 069 [arXiv:2301.12922]

$$\rho_{\phi\phi} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & |w_{-1\,-1}|^2 & 0 & w_{-1\,-1}w^*_{0\,0} & 0 & w_{-1\,-1}w^*_{1\,1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{0\,0}w^*_{-1,-1} & 0 & |w_{0\,0}|^2 & 0 & w_{0\,0}w^*_{1\,1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{1\,1}w^*_{-1\,-1} & 0 & w_{1\,1}w^*_{0\,0} & 0 & |w_{1\,1}|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Entropy

$$\mathscr{E}[\rho] = 0.531 \pm 0.040$$ **(13,3 σ)**

CGLMP $\mathcal{I}_3$

$$\text{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.296 \pm 0.034$$ **(8,8 σ)**

✔

Charmonium spin-1 states

Qubits (spin ½)

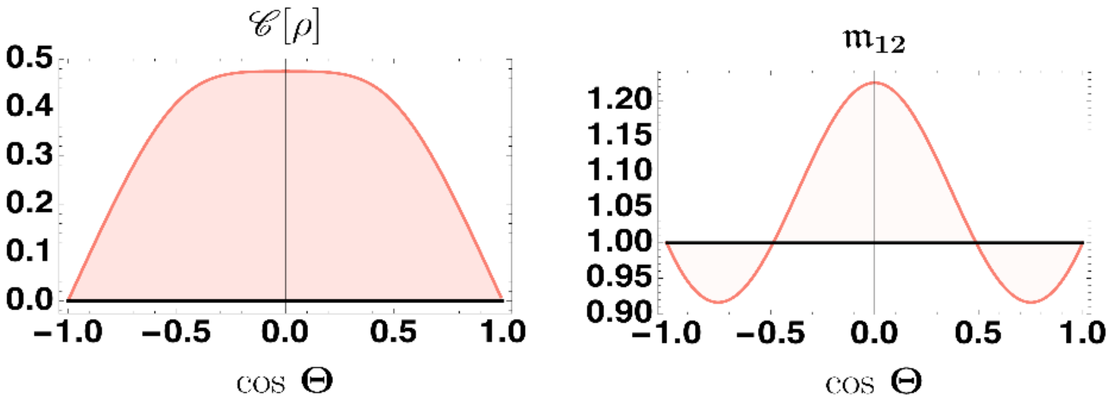
$$J/\psi \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}$$

$$|\psi_\uparrow\rangle \quad \propto \quad w_{\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\;\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\;\tfrac{1}{2}\rangle$$

$$|\psi_\downarrow\rangle \quad \propto \quad w_{-\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\;-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\;-\tfrac{1}{2}\rangle$$

$$|\psi_0\rangle \quad \propto \quad w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2}\;\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\;-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2}\;-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\;\tfrac{1}{2}\rangle$$

$$w_{\frac{1}{2}\,\frac{1}{2}} = w_{-\frac{1}{2}\,-\frac{1}{2}} = \frac{\sqrt{1-\alpha}}{\sqrt{2}} \qquad \text{and} \qquad w_{\frac{1}{2}\,-\frac{1}{2}} = w_{-\frac{1}{2}\,\frac{1}{2}} = \sqrt{1+\alpha}\,\exp[-i\Delta\Phi]$$



analysis based on
10bilions J/Psi events
@ BesIII experiment
3.2M $\Lambda\Lambda$ events expected

$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}}$$

$$\Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}}$$

BesIII Collaboration, Phys. Rev. Lett 129 (2022) n. 13 131801 [arXiv:2204.11058]

Concurrence

$\mathscr{C}[\rho] = 0.475 \pm 0.004$ **(118,7σ)**

Horodecki condition

$\mathfrak{m}_{12} = 1.225 \pm 0.004$ **(56,3σ)**

✔

Bell inequality violation

decay	$\mathfrak{m}_{12}$	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 110 (2024) 3, 053008

ongoing work

$pp \to t\bar{t}$

A

LHC, data already available

Analysis under way

$pp \to H \to ZZ^*$

B

LHC, data already available

Analysis under way

$e^+e^- \to \tau^+\tau^-$

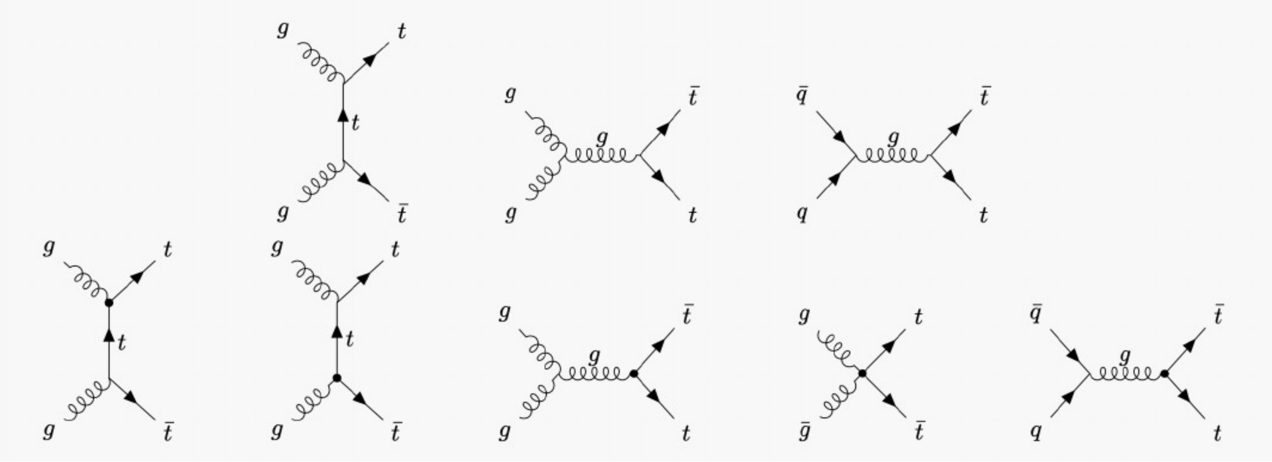
C

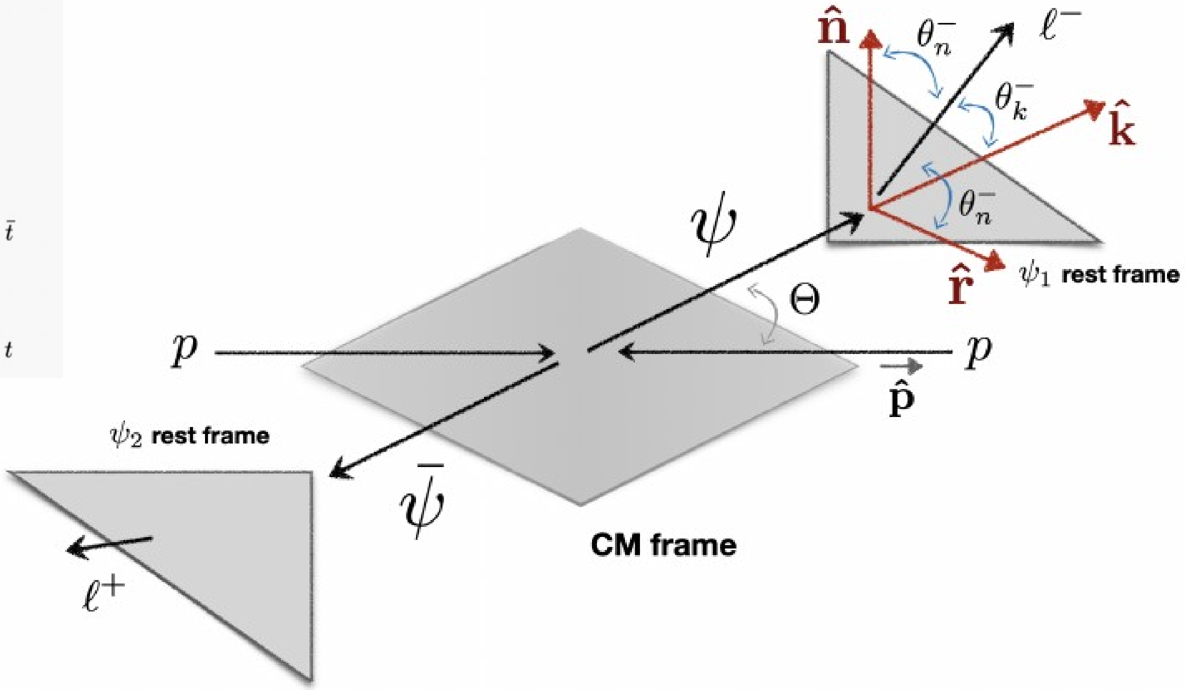
Belle II, data already available

Analysis under way

While waiting — let us see some simulations

Bell inequality violation in top-antitop production at LHC





$C_{ab}(m_{t\bar{t}}, \Theta) \longrightarrow$ can be extracted by fitting the double angle distribution

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_+^{\mathrm{a}}\ \mathrm{d}\cos\theta_-^{\mathrm{b}}} = \frac{1}{4}\left(1 + C_{ab}\ \cos\theta_+^{\mathrm{a}}\ \ \cos\theta_-^{\mathrm{b}}\right)$$

$$\hat{\mathbf{n}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} \times \hat{\mathbf{k}}\right)$$

angles computed in the corresponding rest frame of the decaying top or antitop

$$a \text{ and } b \in \{\mathrm{k}, \mathrm{n}, \mathrm{r}\}$$

$$\cos\theta_-^b = \hat{\ell}_- \cdot \hat{b}$$

$$\cos\theta_+^a = \hat{\ell}_+ \cdot \hat{a}$$

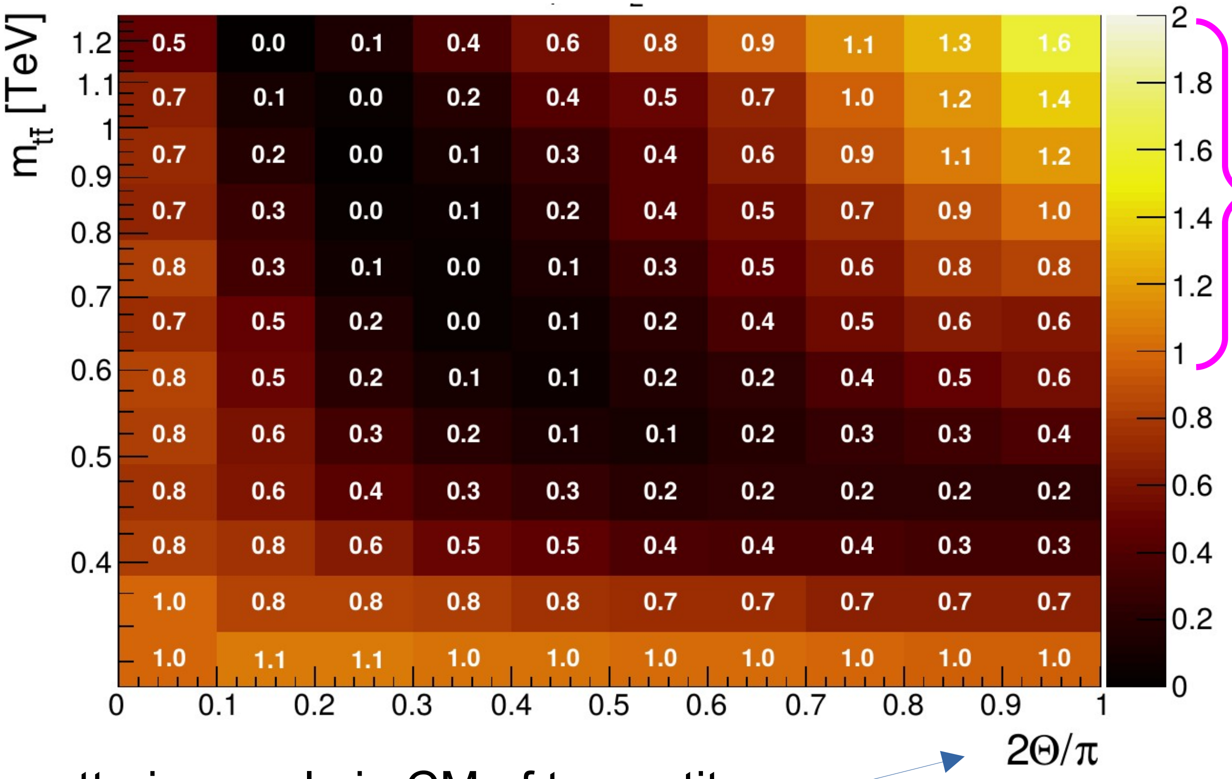
$$\hat{\mathbf{r}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} - \cos\Theta\hat{\mathbf{k}}\right)$$

Montecarlo simulations

M. Fabbrichesi, R. Floreanini, G. Panizzo, Phys. Rev. Letters 127 (2021), 2102.11883 [hep-ph]

invariant mass of top-antitop

$\mathfrak{m}_{12}[C]$



0.5	0.0	0.1	0.4	0.6	0.8	0.9	1.1	1.3	1.6
0.7	0.1	0.0	0.2	0.4	0.5	0.7	1.0	1.2	1.4
0.7	0.2	0.0	0.1	0.3	0.4	0.6	0.9	1.1	1.2
0.7	0.3	0.0	0.1	0.2	0.4	0.5	0.7	0.9	1.0
0.8	0.3	0.1	0.0	0.1	0.3	0.5	0.6	0.8	0.8
0.7	0.5	0.2	0.0	0.1	0.2	0.4	0.5	0.6	0.6
0.8	0.5	0.2	0.1	0.1	0.2	0.2	0.4	0.5	0.6
0.8	0.6	0.3	0.2	0.1	0.1	0.2	0.3	0.3	0.4
0.8	0.6	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.8	0.8	0.6	0.5	0.5	0.4	0.4	0.4	0.3	0.3
1.0	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.7
1.0	1.1	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0

scattering angle in CM of top-antitop

First analysis of Bell inequalities where correlation matrix C_{ij} is extracted from event simulation, including ATLAS detector resolution (Delphes), acceptance, migration and efficiency effects.

Bell inequality violation
Horodecki condition

$$\mathfrak{m}_{12}[C] > 1$$

Violation of null hypothesis can be assessed:

- at 2 σ level with present Run 2 Luminosity
- at 4 σ with projected full Run 3 Luminosity

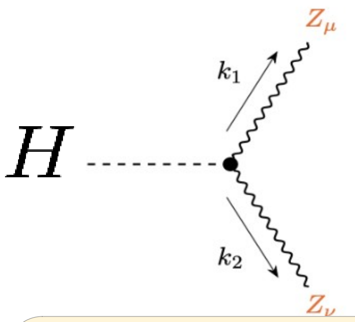
$$\mathcal{L}_{\text{dipole}} = \frac{c_{tG}}{\Lambda^2}\left(\mathcal{O}_{tG} + \mathcal{O}^{\dagger}_{tG}\right) \quad \text{with} \quad \mathcal{O}_{tG} = g_s\,(\bar{Q}_L\,\sigma^{\mu\nu}\,T^a\,t_R)\,\tilde{H}G^a_{\mu\nu}$$

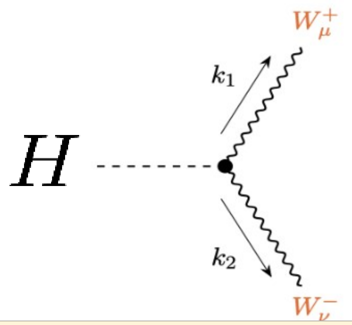
Sensitivity to NP (EFT) studied in

R. Aoude, E. Madge, F. Maltoni, L. Mantani, Phys. Rev. D 106 (2022) 5, 055007; [arXiv:2203.05619]
C. Severi, E. Vryonidou, JHEP 01 (2023) 148; [arXiv:2210.09339]

M. Fabbrichesi, EG, R. Floreanini, EPJC 83 (2023) 2,162; [arXiv:2208.11723]

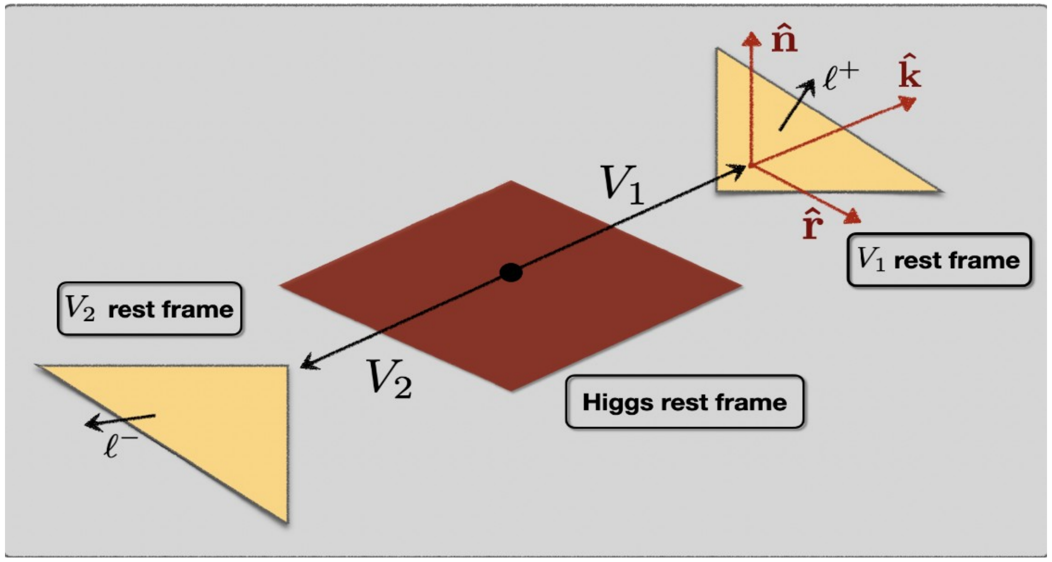
Entanglement and Bell inequality violation in Higgs → ZZ*





$$H \to V(k_1, \lambda_1)\, V^*(k_2, \lambda_2)$$

V=Z,W



$$|\Psi\rangle = \frac{1}{\sqrt{2+\varkappa^2}} \left[|{+}{-}\rangle - \varkappa\, |0\,0\rangle + |{-}{+}\rangle \right]$$

$$\varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

$$M_V^* = f M_V$$

maximum entanglement for χ=1 (ZZ* both at rest)

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\rho_H = |\Psi\rangle\langle\Psi|$$

$$\rho_H^2 = \rho_H$$ Pure state

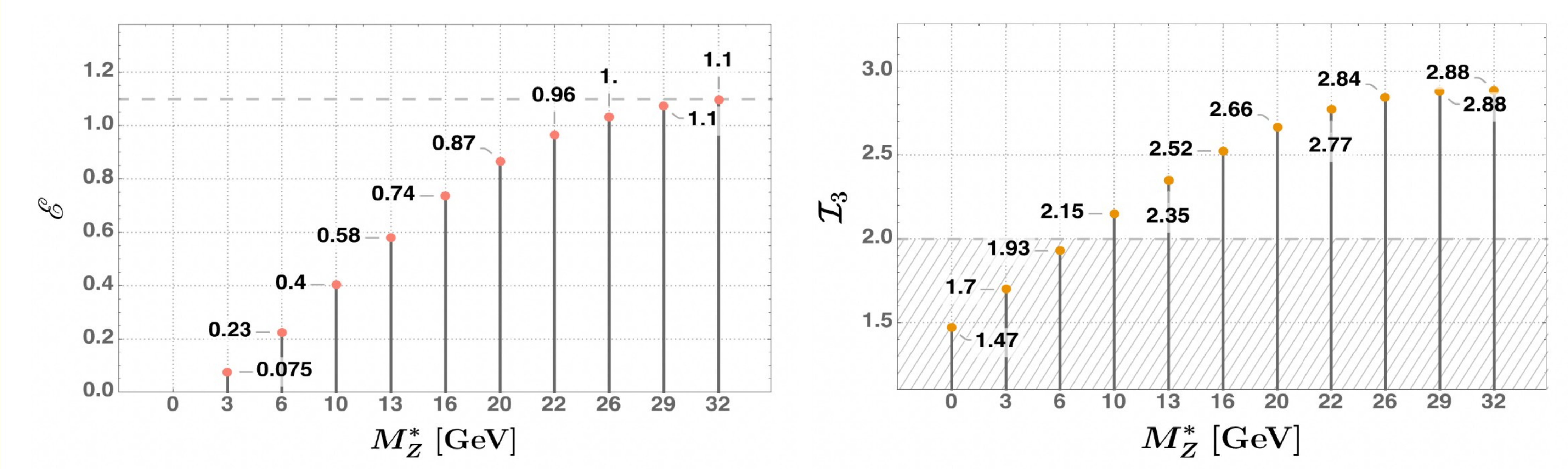
M. Fabbrichesi, EG, R. Floreanini, L. Marzola EPJC 83 (2023) 9,823; [arXiv:2302.00683]

$H \to Z\ Z^*$ SM expectations

M. Fabbrichesi, EG, R. Floreanini, L. Marzola EPJC 83 (2023) 9,823

Quantum entanglement (Entropy)

Bell inequality violation ($\mathcal{I}_3$ >2)



Montecarlo simulation

MadGraph5_aMC@NLO Luminosity of 3ab^{-1} (Hi-Lumi at LHC)

Expected significance for observing the Bell inequality violation is 4.5σ

J.A. Aguilar-Saavedra, A. Bernal, J.A. Casas, J.M. Moreno, Phys . Rev. D 107 1 (2023) 016012, [arXiv:2209.13441] → tensor basis

R. Ashby-Pickering, A.J. Barr, A. Wierzchucka, JHEP 05 (2023) 020; [arXiv:2209.13990] → Gell-Mann basis

Entanglement and quantum tomography at work for Higgs anomalous couplings

$$\mathcal{L}_{HVV} = g\,M_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} M_Z Z_\mu Z^\mu H - \frac{g}{M_W}\left[\frac{a_W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \frac{\widetilde{a}_W}{2} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} + \frac{a_Z}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\widetilde{a}_Z}{4} Z_{\mu\nu} \widetilde{Z}^{\mu\nu}\right] H$$

$$\mathscr{E}_{ent} = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

$$\mathscr{C}_2$$

$$\mathscr{C}_{odd} = \frac{1}{2} \sum_{\substack{a,b \\ a<b}} \left| h_{ab} - h_{ba} \right|$$

⇨ **3 observables**

$$\sum_i \left[\frac{O_i(a_V, \widetilde{a}_V) - O_i(0,0)}{\sigma_i}\right]^2 \leq 5.991$$

χ^2 test with 3 dof

@95C.L.

dedicated Montecarlo to estimate uncertainties

$$f_{g2} = \frac{\sigma_2}{\sigma}\,|a_V|^2\,, \quad \text{and} \quad f_{g3} = \frac{\sigma_3}{\sigma}\,|\widetilde{a}_V|^2$$

ours $f^Z_{g2} < 7.8 \times 10^{-6}\,, \quad f^Z_{g3} < 1.5 \times 10^{-5}$

CMS $f^V_{g2} < 3.4 \times 10^{-3}\,, \quad f^V_{g3} < 1.4 \times 10^{-2}$

@ 95%C.L.

our limits are mostly idealized whereas CMS includes statistical, systematics uncertainties+ background

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, JHEP 09 (2023) 195; [arXiv:2304.02403]

Entanglement and Bell inequality violation at Belle II

$$e^+ + e^- \to \tau^- + \tau^+$$

$\sqrt{s} = 10$ GeV at SuperKEKB

$$\mathscr{C}[\rho] = \frac{(s - 4\,m_\tau^2)\sin^2\Theta}{4\,m_\tau^2 \sin^2\Theta + s\,(\cos^2\Theta + 1)}$$

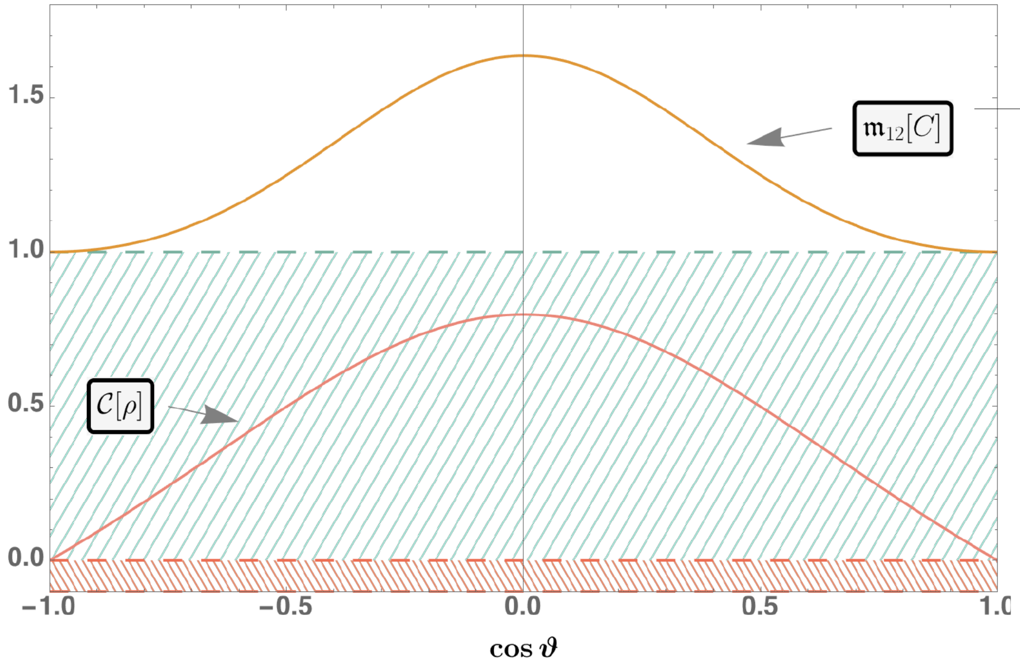
$$\rho_{\tau\bar{\tau}} = \lambda\rho^{(+)} + (1-\lambda)\,\rho^{(1)}_{\text{mix}} \quad \text{with} \quad \lambda = \frac{\beta_\tau^2}{2-\beta_\tau^2}$$

$$\tilde{\rho}^{(2)}_{\text{mix}} = \frac{1}{2}\Big(|\text{RR}\rangle\langle\text{RR}| + |\text{LL}\rangle\langle\text{LL}|\Big)$$

$$\tilde{\rho}^{(+)} = |\tilde{\psi}^{(+)}\rangle\langle\tilde{\psi}^{(+)}|\,, \qquad |\tilde{\psi}^{(+)}\rangle = \frac{1}{\sqrt{2}}\Big(|+-\rangle + |-+\rangle\Big)$$

$$\mathfrak{m}_{12} = 1 + \left(\frac{(s - 4\,m_\tau^2)\sin^2\Theta}{4\,m_\tau^2 \sin^2\Theta + s\,(\cos^2\Theta + 1)}\right)^2$$

- at threshold $\beta_\tau \simeq 0$ the state is a mixed one, with no quantum correlations
- at relativistic regime $\beta_\tau \to 1$ the state is maximally entangled



Bell inequality violation

Entanglement

$e^+ + e^- \to \tau^- + \tau^+$ Montecarlo simulations for Belle II

Assuming data set of about 200million of events. Analysis based on six decay channels

$$\pi^+\pi^-,\ \pi^\pm\rho^\mp,\ \pi^\pm a_1^\mp,\ \rho^+\rho^-,\ \rho^\pm a_1^\mp\ \ a_1^+a_1^-$$

Spin orientation reconstructed using the polarimeter vector method

S. Jadach, J. H. Kühn, and Z. Was, "TAUOLA: a library of Monte Carlo programs to simulate decays of polarized tau leptons," *Comput. Phys. Commun.* **64** (1990) 275.

V. Cherepanov and C. Veelken, "The polarimeter vector for $\tau \to 3\pi\nu_\tau$ decays," `arXiv:2311.10490 [hep-ex]`.

Decay channel	$\mathcal{C}[\rho]$	$\mathfrak{m}_{12}[\mathbf{C}]$
$\pi^+\pi^-$	0.7079 ± 0.0071	1.483 ± 0.011
$\pi^\pm\rho^\mp$	0.7113 ± 0.0029	1.482 ± 0.008
$\pi^\pm a_1^\mp$	0.6762 ± 0.0028	1.388 ± 0.009
$\rho^+\rho^-$	0.7111 ± 0.0032	1.495 ± 0.007
$\rho^\pm a_1^\mp$	0.6798 ± 0.0026	1.402 ± 0.008
$a_1^+a_1^-$	0.6386 ± 0.0060	1.294 ± 0.018
All channels	0.6905 ± 0.0014	1.444 ± 0.004

Events passing selection cuts $|\cos(\vartheta)| < 0.40$

Observation of Quantum entanglement and Bell inequality violation expected with a significance well above 5σ

K. Ehataht, M. Fabbrichesi, L. Marzola, C. Veelken, Phys. Rev. D. 109 (2024) 3, 032005; [arXiv: 2311.17555]

Entanglement at work for New Physics search at Belle II

$$\Gamma^\mu(\tau) = \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\tau} F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau} F_3(q^2)\right]$$

→ EM tau-vertex

$$a_\tau = F_2(0) \quad \text{and} \quad d_\tau = \frac{e}{2m_\tau} F_3(0)$$

$$\mathcal{L} = e[\bar{\tau}\Gamma^\mu \tau] A_\mu$$

- NP can arise from the following 3 contact-interactions (CI) **dim. 5 operators**

$$\hat{O}_1 = e\frac{c_1}{m_\tau^2}\bar{\tau}\gamma^\mu \tau D^\nu F_{\mu\nu}$$

$$\hat{O}_2 = e\frac{c_2\, v}{2m_\tau^2}\bar{\tau}\sigma^{\mu\nu}\tau F_{\mu\nu}$$

$$\hat{O}_3 = e\frac{c_3\, v}{2m_\tau^2}\bar{\tau}\sigma^{\mu\nu}\gamma_5 \tau F_{\mu\nu}$$

$$F_1(q^2) = 1 + c_1 \frac{q^2}{m_\tau^2} + \dots$$

$$F_{2,3}(0) = 2\, c_{2,3}\frac{v}{m_\tau}$$

- Three observables $\mathscr{O}_i(a_\tau, d_\tau, c_1)$ employed to constrain NP

$$\mathscr{C}_{odd} = \frac{1}{2}\sum_{\substack{i,j\\ i<j}} \left| \mathrm{C}_{ij} - \mathrm{C}_{ji} \right|$$

Concurrence $\mathscr{C}[\rho]$

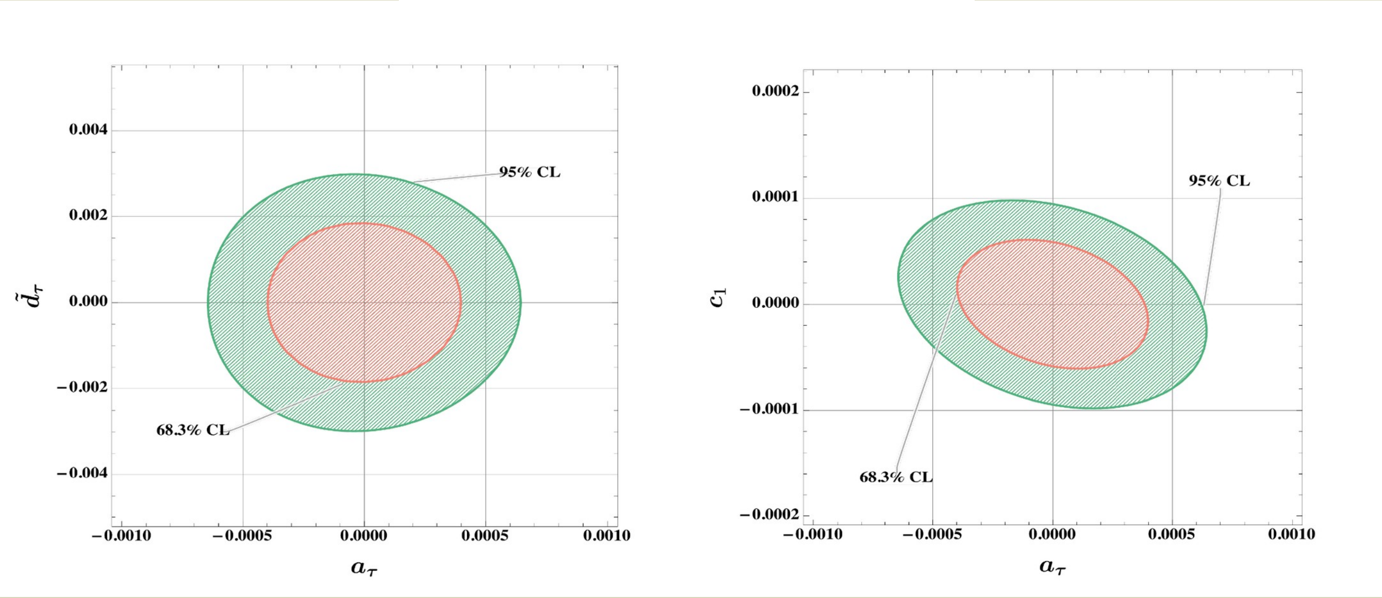
Total cross section

Entanglement at work for New Physics search at Belle II

$$\sum_i \left[\frac{\mathscr{O}_i(a_\tau, d_\tau, c_1) - \mathscr{O}_i(0,0,0)}{\sigma_i}\right]^2 \leq (2.30)\ 5.99$$

χ^2 test with 3 dof

$$e^+e^- \to \tau^+\tau^-$$



benchmark
Luminosity 1ab^{-1}

$$a_\tau^{\mathrm{SM}} = 1.17721(5) \times 10^{-3}$$

PDG (2022)	*This work*
$-1.9 \times 10^{-17} \leq d_\tau \leq 6.1 \times 10^{-18}$ e cm	$\lvert d_\tau \rvert \leq 1.7 \times 10^{-17}$ e cm
$-5.2 \times 10^{-2} \leq a_\tau \leq 1.3 \times 10^{-2}$	$\lvert a_\tau \rvert \leq 6.3 \times 10^{-4}$
$\Lambda_{\mathrm{C.I.}} \geq 7.9$ TeV	$\lvert c_1 \rvert \leq 9.5 \times 10^{-5}$, $\Lambda_{\mathrm{C.I.}} \geq 2.6$ TeV

Limits @ 95% C.L.

M. Fabbrichesi, L. Marzola, Phys. Rev. D 109 (2024) 9, 095026; [arXiv:2401.04449]

K. Ehataht M. Fabbrichesi, L. Marzola, C. Veelken, Phys. Rev. D. 109 (2024) 3, 032005; [arXiv: 2311.17555]

Backup slides

Closing the locality loophole (LL)

LL-1

- One must consider decays in which the produced particles are identical as in the **B** → ϕ ϕ

  (so their life time is also the same)

we need to check how many events satisfies the space-like condition

$$\frac{|t_1 - t_2|\, c}{(t_1 + t_2)\, v} < 1$$

$t_{1,2}$ → time of decays

decay times follow the PDF distribution P(t) ~ Exp[- γ β t]

β → the velocity in unit of c

γ → the Lorentz factor

- For **B** → ϕ ϕ we find that almost 90% of events satisfies this condition
- the two bases used in measuring the polarization are arbitrarily chosen (U V diagonalization)
- → provides a set-up where orientations of polarimeters can be freely and arbitrarily chosen

- So locality loophole can be closed!

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104

EG and L. Marzola, Symmetry 6 (2024) 8, 1036

Closing the detection loophole (DL)

- DL exploits the fact that detectors are not 100% efficient
- Already for qubit the DL is closed if efficiency is more than 80%
- This requirement above is even lower for states belonging to larger Hilbert space as qutrits
- The efficiency of LHCb detector for pion, Kaons, and muons is more than 90%

- So also detection loophole is closed for LHCb !

How to Extract Density Matrix of Two-Qubits from data

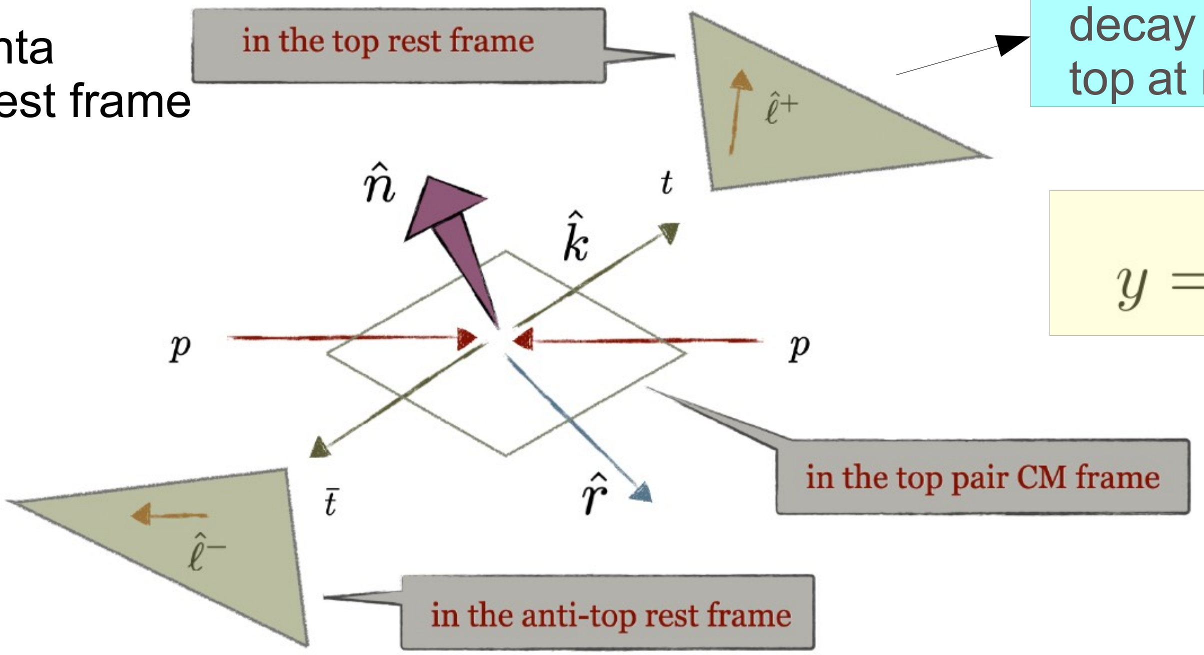
DM-1

Consider pp production of top anti-top system (helicity base)

right-handed basis $\left\{ \hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}} \right\}$ $\hat{\mathbf{n}} = \hat{\mathbf{r}} \times \hat{\mathbf{k}}$ spin-quantization axis of top

lepton(+) momenta direction in top rest frame

decay plane of top at rest



Look at

$$\cos\theta^a_+ = \hat\ell_+ \cdot \hat a$$

$$\cos\theta^b_- = \hat\ell_- \cdot \hat b$$

a and $b \in \{\mathrm{k}, \mathrm{n}, \mathrm{r}\}$

$$y = \hat p \cdot \hat k = \cos\Theta$$

top scattering-angle in c.m. top anti-top

$$\hat p = (0,0,1), \quad \hat r = \frac{1}{r}(\hat p - y\hat k), \quad \hat n = \frac{1}{r}(\hat p \times \hat k)$$

$$\xi_{ab} = \cos\theta^a_+ \cos\theta^b_-$$

For example for specific axes

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4}\Big(1 + B_1 \cos\theta_+ + B_2 \cos\theta_- - C \cos\theta_+ \cos\theta_-\Big)$$

$$C_{ab}\big[\sigma(m_{t\bar t}, \cos\Theta)\big] = -9\,\frac{1}{\sigma}\int d\xi_{ab}\frac{d\sigma}{d\xi_{ab}}\xi_{ab}$$

average of ξ_{ab}

Bernreuther, Heisler, Si , JHEP 12 (2015); arXiv:1508.05271 [hep-ph]

How to Extract Density Matrix of Two-Qutrits from data

DM-2

W W

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^+\,\mathrm{d}\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1V_2}\left(\Gamma_+ \otimes \Gamma_-\right)\right]$$

depend on the invariant mass m_{VV} (or velocity β) and scattering angle Θ in the V_1V_2 cm frame

R. Rahaman, R.K. Singh, NPB 984 (2022) 115984, [arXiv:2109.09345]

$$\mathrm{d}\Omega^\pm = \sin\theta^\pm \mathrm{d}\theta^\pm\, \mathrm{d}\phi^\pm \longrightarrow$$

polar angle $\ell^\pm$ azimuthal angle $\ell^\pm$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1V_2}$ = density matrix of V_1V_2

$\Gamma_\pm \longrightarrow$ Density matrices that describe the polarization of the two decaying W into final leptons (the charged ones assumed to be massless)

these are projectors in the case of the W-bosons because of their chiral couplings to leptons

R. Ashby-Pickering, A.J. Barr, A. Wierzchucka, JHEP 05 (2023) 020; [arXiv:2209.13990] can be computed by rotating to an arbitrary polar axis the spin states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_\pm = \frac{1}{3}\mathbb{1} + \sum_{i=1}^{8} \mathfrak{q}_\pm^a\, T^a$$

→ Density matrices for W-bosons

$\mathfrak{q}_\pm^a$ (Wigner q-symbols) are functions of the corresponding spherical coordinates

↓ set of polynomials of spherical coordindates (see backup slide)

$$\begin{aligned} h_{ab} &= \frac{1}{\sigma}\int\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^+\,\mathrm{d}\Omega^-}\,\mathfrak{p}_+^a\,\mathfrak{p}_-^b\,\mathrm{d}\Omega^+\mathrm{d}\Omega^- \\ f_a &= \frac{1}{\sigma}\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^+}\,\mathfrak{p}_+^a\,\mathrm{d}\Omega^+ \\ g_a &= \frac{1}{\sigma}\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^-}\,\mathfrak{p}_-^a\,\mathrm{d}\Omega^- \end{aligned}$$

$\mathfrak{p}_\pm^n$ a particular set of orthogonal functions (see next slide) ⇨ $\left(\frac{3}{4\,\pi}\right)\int \mathfrak{p}_\pm^n\,\mathfrak{q}_\pm^m\,\mathrm{d}\Omega^\pm = \delta^{nm}$

For **ZZ** case, the set of functions are linear combinations of $\mathfrak{q}_\pm^a$ → see backup slides

Wigner's *Q symbols*

$$
\begin{aligned}
\mathfrak{q}^1_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(\cos\theta^\pm\pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{q}^2_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(\cos\theta^\pm\pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{q}^3_\pm &= \frac{1}{8}\Big(1\pm 4\cos\theta^\pm+3\cos 2\theta^\pm\Big)\,,\\
\mathfrak{q}^4_\pm &= \frac{1}{2}\sin^2\theta^\pm\cos 2\,\phi^\pm\,,\\
\mathfrak{q}^5_\pm &= \frac{1}{2}\sin^2\theta^\pm\sin 2\,\phi^\pm\,,\\
\mathfrak{q}^6_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(-\cos\theta^\pm\pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{q}^7_\pm &= \frac{1}{\sqrt{2}}\sin\theta^\pm\Big(-\cos\theta^\pm\pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{q}^8_\pm &= \frac{1}{8\sqrt{3}}\Big(-1\pm 12\cos\theta^\pm-3\cos 2\theta^\pm\Big)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{p}^1_\pm &= \sqrt{2}\sin\theta^\pm\Big(5\cos\theta^\pm\pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{p}^2_\pm &= \sqrt{2}\sin\theta^\pm\Big(5\cos\theta^\pm\pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{p}^3_\pm &= \frac{1}{4}\Big(5\pm 4\cos\theta^\pm+15\cos 2\theta^\pm\Big)\,,\\
\mathfrak{p}^4_\pm &= 5\sin^2\theta^\pm\cos 2\,\phi^\pm\,,\\
\mathfrak{p}^5_\pm &= 5\sin^2\theta^\pm\sin 2\,\phi^\pm\,,\\
\mathfrak{p}^6_\pm &= \sqrt{2}\sin\theta^\pm\Big(-5\cos\theta^\pm\pm 1\Big)\cos\phi^\pm\,,\\
\mathfrak{p}^7_\pm &= \sqrt{2}\sin\theta^\pm\Big(-5\cos\theta^\pm\pm 1\Big)\sin\phi^\pm\,,\\
\mathfrak{p}^8_\pm &= \frac{1}{4\sqrt{3}}\Big(-5\pm 12\cos\theta^\pm-15\cos 2\theta^\pm\Big)\,.
\end{aligned}
$$

Bell inequality

follows from slide 9

Some definitions

Consider the measurements on three directions **a,b,c** **(not necessarily orthogonal)** on a N sample of particles pairs (particles1 and particles2)

Particle 1

$$N_1 \qquad (\hat{\mathbf{a}}+,\hat{\mathbf{b}}+,\hat{\mathbf{c}}+)$$

N_1 subsample of particles that if we measure spin in **a** direction we find with certainty + and in **b** direction + and in **c** direction +

Particle 2

$$N_1 \qquad (\hat{\mathbf{a}}-,\hat{\mathbf{b}}-,\hat{\mathbf{c}}-)$$

due to angular momentum conservation

BI-2

- separate the two particles and measure spin correlations

Spin-correlations as predicted by deterministic (causal) local theories

Population	Particle 1	Particle 2
N_1	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$
N_2	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$
N_3	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_4	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$
N_5	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$
N_6	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$
N_7	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_8	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$

8 possible combinations

Since each $N_{1\text{-}8}$ is semipositive defined

$$N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7)$$

Probability

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) = \frac{(N_3 + N_4)}{\sum_i^8 N_i}$$

spin+ along **b** for particle 2

spin+ along **a** for particle 1

$$P(\hat{\mathbf{c}}+; \hat{\mathbf{b}}+) = \frac{(N_3 + N_7)}{\sum_i^8 N_i} \qquad P(\hat{\mathbf{a}}+; \hat{\mathbf{c}}+) = \frac{(N_2 + N_4)}{\sum_i^8 N_i}$$

Bell inequality

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) \leq P(\hat{\mathbf{a}}+; \hat{\mathbf{c}}+) + P(\hat{\mathbf{c}}+; \hat{\mathbf{b}}+)$$

the massless case is analogous of two entangled photons

$$\Psi = \frac{1}{\sqrt{2}}\left(|\gamma_R^l\rangle|\gamma_L^r\rangle + |\gamma_L^l\rangle|\gamma_R^r\rangle\right)$$

L,R → helicities of photon
l,r → coming from left, right directions

In both cases we need a theory to reconstruct polarizations (QED for photons and SM for taus)

Polarization of both massless taus and photons are described by

$$SO(2) \simeq U(1) \text{ which is Abelian.}$$

Non-commutativity nature of the polarizations show up in the four-dimensional space of their polarizations

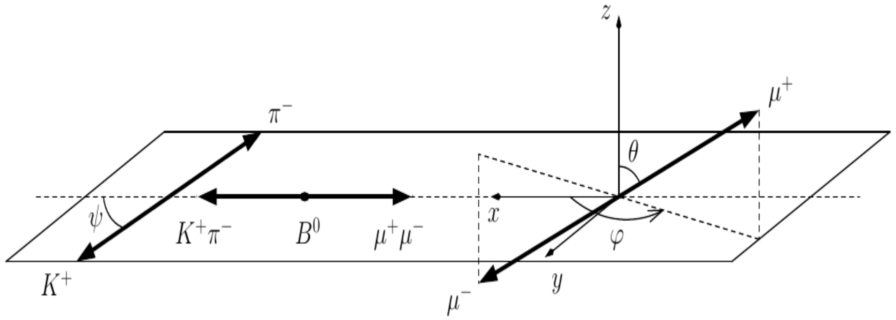
$$\Psi = \frac{1}{\sqrt{2}}\left(|\tau_R^-\rangle|\tau_L^+\rangle + |\tau_L^-\rangle|\tau_R^+\rangle\right) \Rightarrow \Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix} \Rightarrow \begin{bmatrix}\text{RR}\\\text{RL}\\\text{LR}\\\text{LL}\end{bmatrix}$$

$$\frac{1}{2}|\tau_R^-\rangle|\tau_L^+\rangle \quad \text{or} \quad \frac{1}{2}|\tau_L^-\rangle|\tau_R^+\rangle \Rightarrow \Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad \text{or} \quad \Psi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

FIT of coefficients h_k

$$\frac{d^4\Gamma(B^0 \to J/\psi K^{*0})}{dt\,d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega)$$

$$\Omega = \{\cos\theta, \cos\psi, \varphi\}$$



k	h_k	$f_k(\Omega)$
1	$\lvert A_0\rvert^2$	$\frac{9}{32\pi} 2\cos^2\psi(1-\sin^2\theta\cos^2\varphi)$
2	$\lvert A_\parallel\rvert^2$	$\frac{9}{32\pi}\sin^2\psi(1-\sin^2\theta\sin^2\varphi)$
3	$\lvert A_\perp\rvert^2$	$\frac{9}{32\pi}\sin^2\psi\sin^2\theta$
4	$\lvert A_\parallel\rvert\lvert A_\perp\rvert\sin(\delta_\perp-\delta_\parallel)$	$-\frac{9}{32\pi}\sin^2\psi\sin 2\theta\sin\varphi$
5	$\lvert A_0\rvert\lvert A_\parallel\rvert\cos(\delta_\parallel)$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin^2\theta\sin 2\varphi$
6	$\lvert A_0\rvert\lvert A_\perp\rvert\sin(\delta_\perp)$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin 2\theta\cos\varphi$
7	$\lvert A_S\rvert^2$	$\frac{3}{32\pi}2(1-\sin^2\theta\cos^2\varphi)$
8	$\lvert A_\parallel\rvert\lvert A_S\rvert\cos(\delta_\parallel-\delta_S)$	$\frac{3}{32\pi}\sqrt{6}\sin\psi\sin^2\theta\sin 2\varphi$
9	$\lvert A_\perp\rvert\lvert A_S\rvert\sin(\delta_\perp-\delta_S)$	$\frac{3}{32\pi}\sqrt{6}\sin\psi\sin 2\theta\cos\varphi$
10	$\lvert A_0\rvert\lvert A_S\rvert\cos(\delta_S)$	$\frac{3}{32\pi}4\sqrt{3}\cos\psi(1-\sin^2\theta\cos^2\varphi)$

(A_S contribution from non-resonant J/Y K* amplitude)